LECTURE NOTES

ON

THEORY OF MACHINE

DIPLOMA 4th SEMESTER

COMPILED BY

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UNIT 1: Simple Mechanism

Introduction

Mechanics: It is that branch of scientific analysis which deals with motion, time and force.

Kinematics: is the study of motion, without considering the forces which produce that motion. Kinematics of machines deals with the study of the relative motion of machine parts. It involves the study of position, displacement, velocity and acceleration of machine parts.

Dynamics: of machines involves the study of forces acting on the machine parts and the motions resulting from these forces.

Kinematic link (or) element

A machine part or a component of a mechanism is called a kinematic link or simply a link. A link is assumed to be completely rigid, or under the action of forces it does not suffer any deformation, signifying that the distance between any two points on it remains constant. Although all real machine parts are flexible to some degree, it is common practice to assume that deflections are negligible and parts are rigid when analyzing a machine's kinematic performance.

Types of link

(a) Based on number of elements of link:

Binary link: Link which is connected to other links at two points.
Ternary link: Link which is connected to other links at three points
Quaternary link: Link which is connected to other links at four points

In order to transmit motion, the driver and the follower may be connected by the following three types of links:

1. Rigid link. A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

2. Flexible link. A flexible link is one which is partly deformed in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.

3. Fluid link. A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

Machine: A machine is a mechanism or collection of mechanisms, which transmit force from the source of power to the resistance to be overcome. Though all machines are mechanisms, all mechanisms are not machines. Many instruments are mechanisms but are not machines, because they do no useful work nor do they transform energy.

Kinematic pair

The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as kinematic pair.

Classification of kinematic pair

The kinematic pairs may be classified according to the following considerations:

(i) Based on nature of contact between elements:

(a)　Lower pair. If the joint by which two members are connected has surface contact, the pair is known as lower pair. Eg. pin joints, shaft rotating in bush, slider in slider crank mechanism.
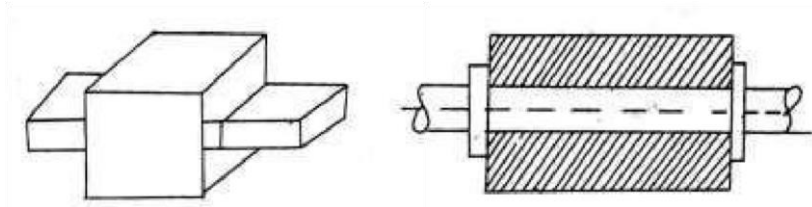

(b)　Higher pair. If the contact between the pairing elements takes place at a point or along a line, such as in a ball bearing or between two gear teeth in contact, it is known as a higher pair.
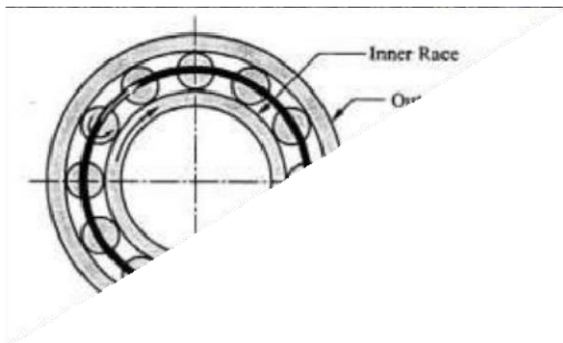

(ii) Based on relative motion between pairing elements:

(a) Siding pair. Sliding pair is constituted by two elements so connected that one is constrained to have a sliding motion relative to the other
(b) Turning pair (revolute pair). When connections of the two elements are such that only a constrained motion of rotation of one element with respect to the other is possible, the pair constitutes a turning pair

(a)
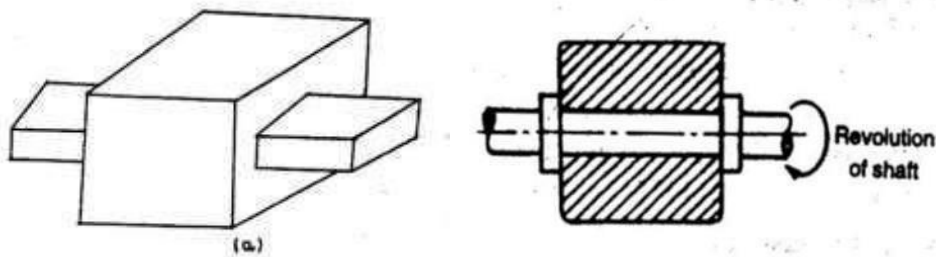
(c) Rolling pair. When the pairing elements have rolling contact, the pair formed is called rolling pair. Eg. Bearings, Belt and pulley

Inner Race
Outer Race
Rolling Element
Cage or Separator
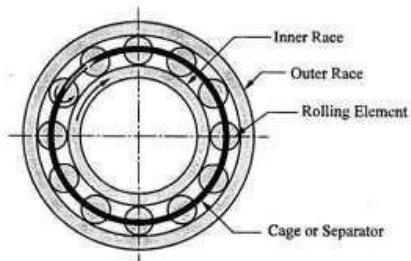
(d) Spherical pair. A spherical pair will have surface contact and three degrees of freedom. Eg. Ball and socket joint.

(e) Helical pair or screw pair. When the nature of contact between the elements of a pair is such that one element can turn about the other by screw threads, it is known as screw pair. Eg. Nut and bolt

(b)

BALL ELEMENT

SOCKET ELEMENT

(c)
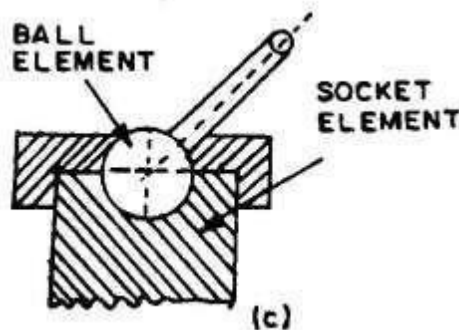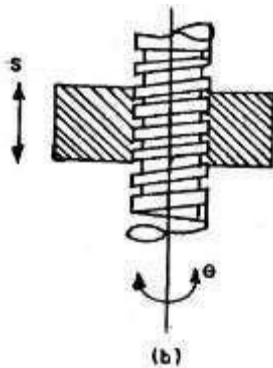
(a) Sliding pair (prismatic pair) eg. piston and cylinder, crosshead and slides, tail stock on lathe bed. (b) Turning pair (Revolute pair): eg: cycle wheel on axle, lathe spindle in head stock.

(c) Cylindrical pair: eg. shaft turning in journal bearing.

(d) Screw pair (Helical pair): eg. bolt and nut, lead screw of lathe with nut, screw jack.

(e) Spherical pair: eg. penholder on stand, castor balls.

Mechanism

When one of the links of a kinematic chain is fixed, the chain is known as mechanism.

A mechanism with four links is known as simple mechanism, and the mechanism with more than four links is known as compound mechanism.

When a mechanism is required to transmit power or to do some particular type of work, it then becomes a machine.

A mechanism is a constrained kinematic chain. This means that the motion of any one link in the kinematic chain will give a definite and predictable motion relative to each of the others. Usually one of the links of the kinematic chain is fixed in a mechanism.
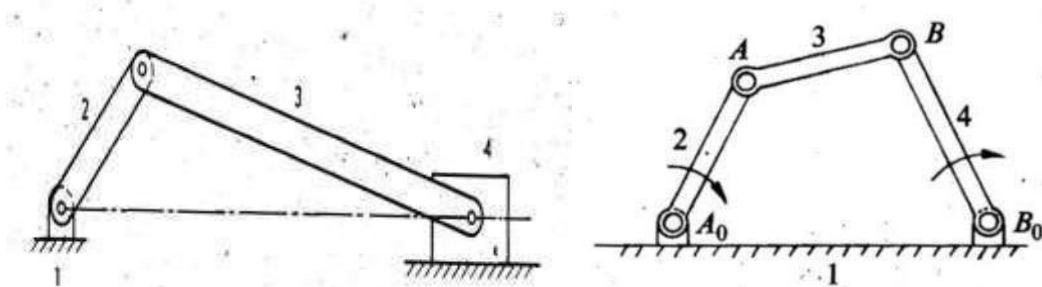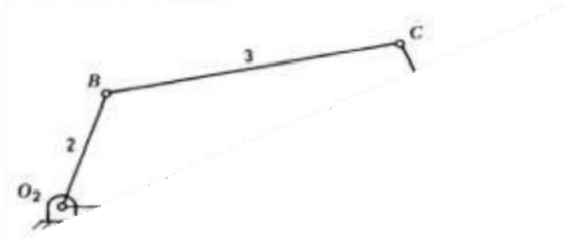

Slider crank and four bar mechanisms
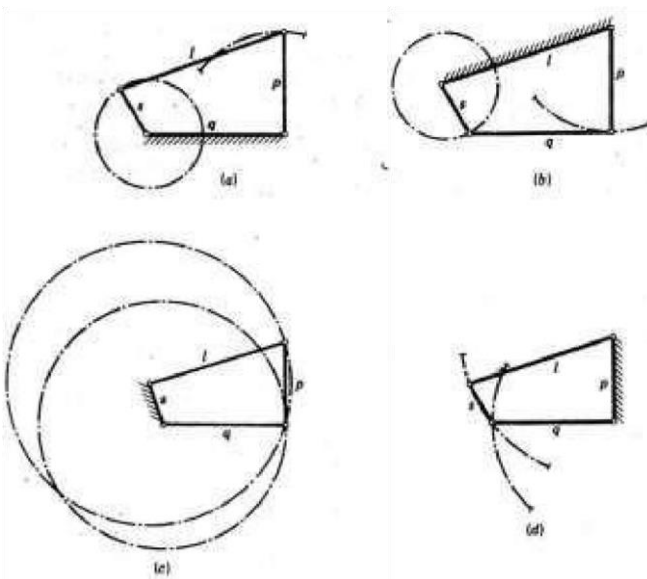

Inversion of Mechanism

A mechanism is one in which one of the links of a kinematic chain is fixed. Different mechanisms can be obtained by fixing different links of the same kinematic chain. These are called as inversions of the mechanism.

Inversions of Four Bar Chain

One of the most useful and most common mechanisms is the four-bar linkage. In this mechanism, the link which can make complete rotation is known as crank (link 2). The link which oscillates is known as rocker or lever (link 4). And the link connecting these two is known as coupler (link 3). Link 1 is the frame.
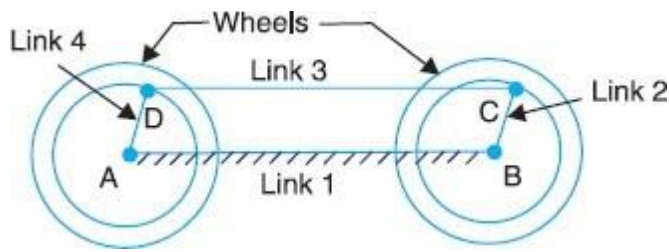
Inversions:

Inversions of four bar chain

Crank-rocker mechanism: In this mechanism, either link 1 or link 3 is fixed. Link 2 (crank) rotates completely and link 4 (rocker) oscillates. It is similar to (a) or (b)

Double crank mechanism (Coupling rod of locomotive). This is one type of drag link mechanism, where, links 1& 3 are equal and parallel and links 2 & 4 are equal and parallel.

Double rocker mechanism. In this mechanism, link 4 is fixed. Link 2 makes complete rotation, whereas links 3 & 4 oscillate

Cam - A mechanical device used to transmit motion to a follower by direct contact. Where Cam – driver member Follower - driven member. The cam and the follower have line contact and constitute a higher pair. In a cam - follower pair, the cam normally rotates at uniform speed by a shaft, while the follower may is predetermined, will translate or oscillate according to the shape of the cam. A familiar example is the camshaft of an automobile engine, where the cams drive the push rods (the followers) to open and close the valves in synchronization with the motion of the pistons.

Applications: The cams are widely used for operating the inlet and exhaust valves of Internal combustion engines, automatic attachment of machineries, paper cutting machines, spinning and weaving textile machineries, feed mechanism of automatic lathes.

Classification of Followers

(i) Based on surface in contact.

(a) Knife edge follower (b) Roller follower (c) Flat faced follower (d) Spherical follower

(a) (b) (c) (d)
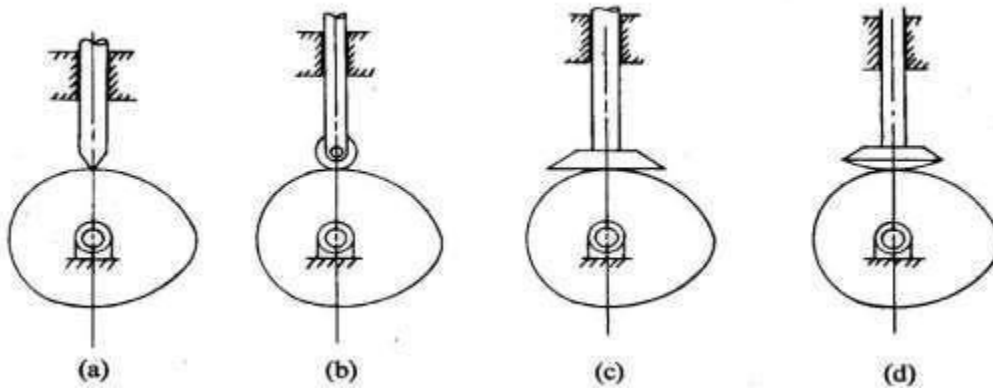
(ii) Based on type of motion:

(a) Oscillating follower

(b) Translating follower

(iii) Based on line of motion:

(a) Radial follower: The lines of movement of in-line cam followers pass through the centers of the camshafts

(b) Off-set follower: For this type, the lines of movement are offset from the centers of the camshafts

Classification of Cams

Cams can be classified based on their physical shape.
a) Disk or plate cam: The disk (or plate) cam has an irregular contour to impart a specific motion to the follower. The follower moves in a plane perpendicular to

the axis of rotation of the camshaft and is held in contact with the cam by springs or gravity.

(a)   (b)
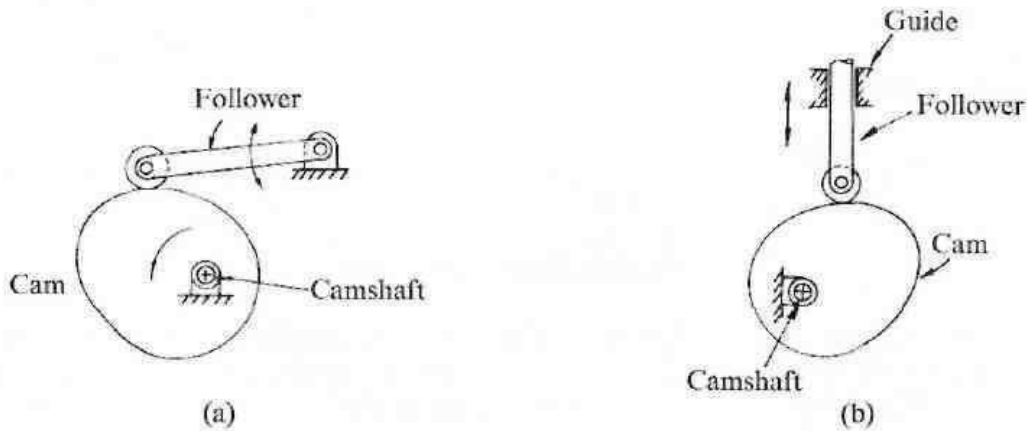
b) Cylindrical cam: The cylindrical cam has a groove cut along its cylindrical surface. The roller follows the groove, and the follower moves in a plane parallel to the axis of rotation of the cylinder
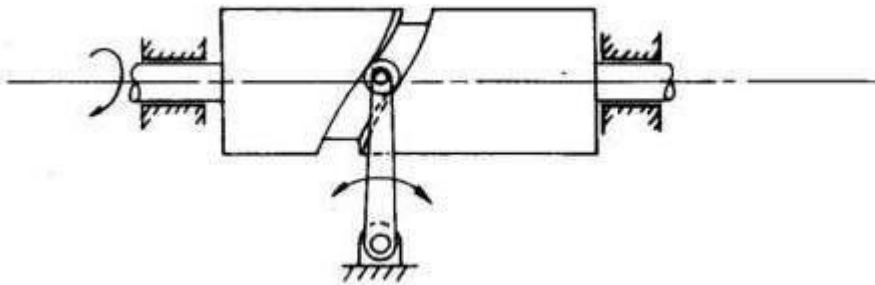
c) Translating cam. The translating cam is a contoured or grooved plate sliding on a guiding surface(s). The follower may oscillate or reciprocate. The contour or the shape of the groove is determined by the specified motion of the follower.

(a)   (b)
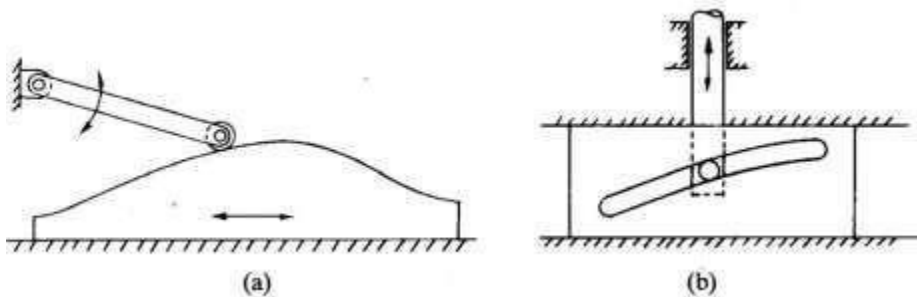
Terms Used in Radial Cams
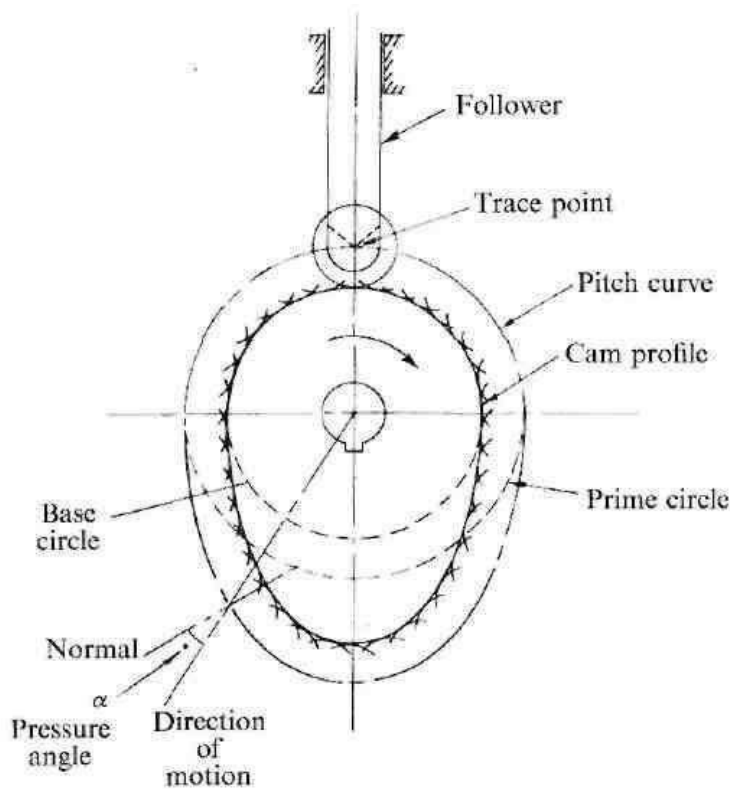
(a)Pressure angle: It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the angle is too large, a reciprocating follower will jam in its bearings.

b) Base circle: It is the smallest circle that can be drawn to the cam profile.
c) Trace point: It is the reference point on the follower and is used to generate the pitch curve. In the case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In the roller follower, the centre of the roller represents the trace point.
d) Pitch point: It is a point on the pitch curve having the maximum pressure angle.
e) Pitch circle: It is a circle drawn from the centre of the cam through the pitch points.
f) Pitch curve: It is the curve generated by the trace point as the follower moves relative to the cam.For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower; they are separated by the radius of the follower.
g) Prime circle: It is the smallest circle that can be drawn from the centre of the cam and tangent to the point. For a knife edge and a flat face follower, the

prime circle and the base circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

h) Lift (or) stroke: It is the maximum travel of the follower from its lowest position to the topmost position.

UNIT 3: Power transmission

Introduction

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds. The amount of power transmitted depends upon the following factors:

1. The velocity of the belt.

2. The tension under which the belt is placed on the pulleys.

3. The conditions under which the belt is used.

Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,

2. Speed reduction ratio,

3. Power to be transmitted,

4. Centre distance between the shafts,

5. Positive drive requirements,

6. Shafts layout,

7. Space available, and
8. Service conditions.

Types of Belt Drives

The belt drives are usually classified into the following three groups:

1. Light drives. These are used to transmit small powers at belt speeds upto about 10 m/s, as in agricultural machines and small machine tools.

2. Medium drives. These are used to transmit medium power at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

3. Heavy drives. These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators.


Types of Belts

(a) Flat belt.   (b) V-belt.   (c) Circular belt.
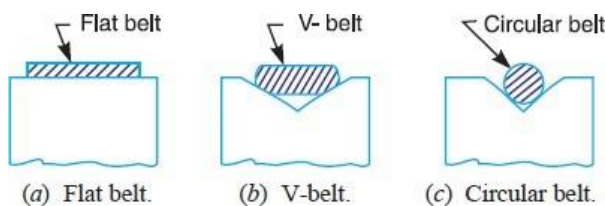
1. Flat belt. The flat belt, as shown in Fig. (a), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.

2. V-belt. The V-belt, as shown in Fig. (b), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

3. Circular belt or rope. The circular belt or rope, as shown in Fig. (c), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart. If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another

Types of Flat Belt Drives

The power from one pulley to another may be transmitted by any of the following types of belt drives: 1. Open belt drive. The open belt drive is used with shafts arranged parallel and rotating in the same direction. In this case, the driver A pulls the belt from one side (i.e. lower side RQ) and delivers it to the other side (i.e. upper side LM). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as tight side whereas the upper side belt (because of less tension) is known as slack side,
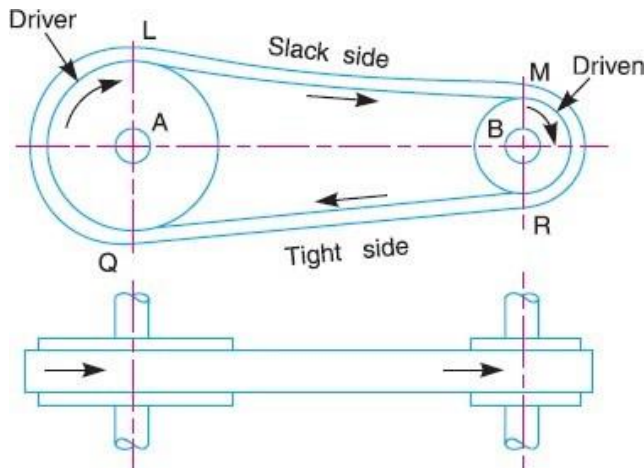
2.   Crossed or twist belt drive. The crossed or twist belt drive is used with shafts arranged parallel and rotating in the opposite directions. In this case, the driver pulls the belt from one side (i.e. RQ) and delivers it to the other side (i.e. LM). Thus the tension in the belt RQ will be more than that in the belt LM. The belt RQ (because of more tension) is known as tight side, whereas the belt LM (because of less tension) is known as slack side,

3.   Quarter turn belt drive. The quarter turn belt drive also known as right angle belt drive, as shown in Fig. (a), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to 1.4 b, where b is the width of belt. In case the pulleys cannot be arranged, as shown in Fig. (a), or when the reversible is desired, then a quarter turn belt drive with guide pulley, as shown in Fig. (b), may be used.
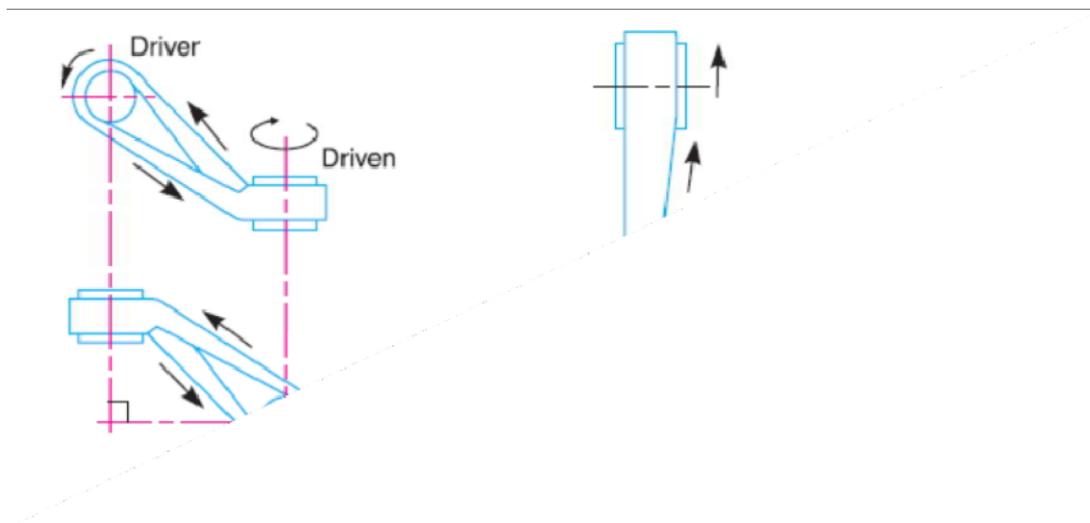
4.　Belt drive with idler pulleys. A belt drive with an idler pulley, as shown in Fig. (a), is used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension cannot be obtained by other means. When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig. (b), may be employed.

(a) Belt drive with single idler pulley.　　(b) Belt drive with many idler pulleys.
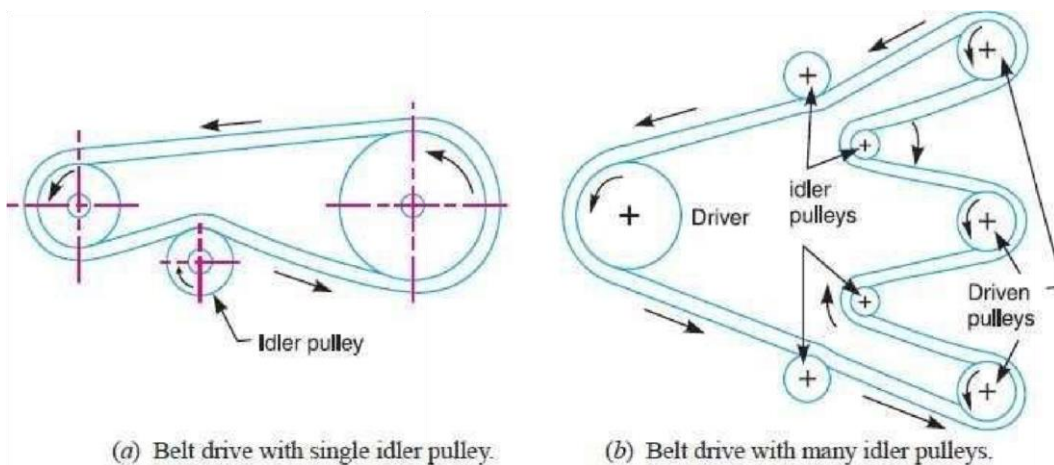
5.　Compound belt drive. A compound belt drive, as shown in Fig., is used when power is transmitted from one shaft to another through a number of pulleys

6. Stepped or cone pulley drive. A stepped or cone pulley drive, as shown in Fig, is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.
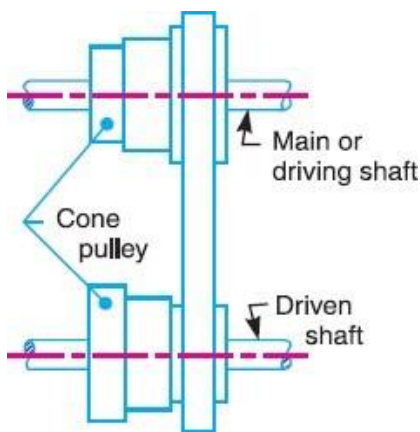
7. Fast and loose pulley drive. A fast and loose pulley drive, as shown in Fig., is used when the driven or machine shaft is to be started or stopped whenever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called fast pulley and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any

power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.
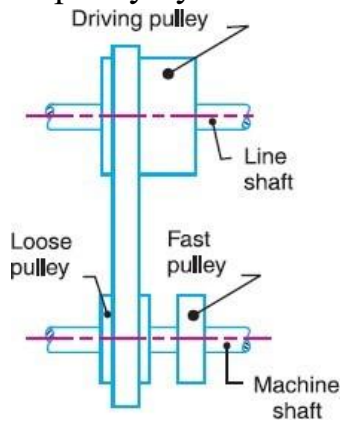
Velocity Ratio of Belt Drive

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below: Let d1 = Diameter of the driver, d2 = Diameter of the follower, N1 = Speed of the driver in r.p.m., and N2 = Speed of the follower in r.p.m.

Length of the belt that passes over the driver, in one minute = π d1N1

Similarly, length of the belt that passes over the follower, in one minute = π d2 N2

Since the length of belt that passes over the driver in one minute is equal to the length of

belt that passes over the follower in one minute, therefore π

d1 N1 = π d2 N2

Velocity ratio, $\dfrac{N_2}{N_1} = \dfrac{d_1}{d_2}$

When the thickness of the belt (t) is considered, then velocity ratio

$$\dfrac{N_2}{N_1} = \dfrac{d_1 + t}{d_2 + t}$$

The velocity ratio of a belt drive may also be obtained as discusse~
We know that peripheral velocity of the belt on the drivin~

$$v_1 = \frac{\pi d_1 . N_1}{60} \; m/s$$

and peripheral velocity of the belt on the driv~

When there i~

Velocity Ratio of a Compound Belt Drive Sometimes the power is transmitted from one shaft to another, through a number of pulleys, as shown in fig. Consider a pulley 1 driving the pulley 2. Since the pulleys 2 and 3 are keyed to the same shaft, therefore the pulley 1 also drives the pulley 3 which, in turn, drives the pulley 4.

Let
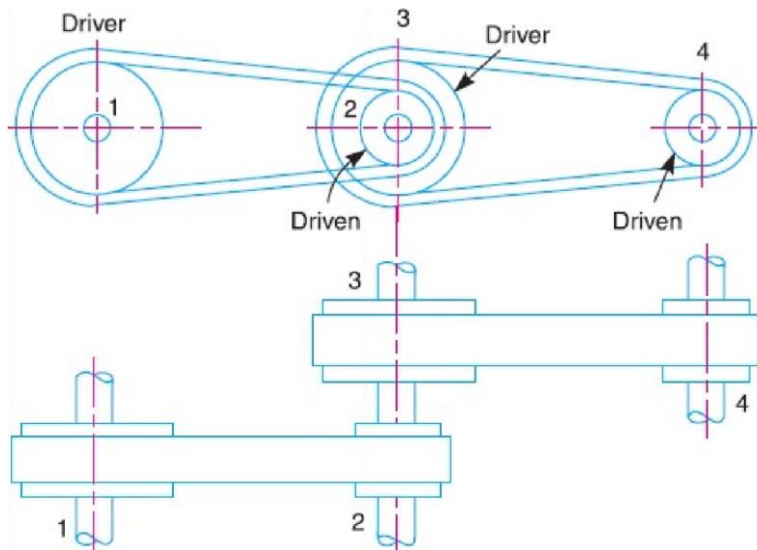
d1 = Diameter of the pulley 1, N1 = Speed of the pulley 1 in r.p.m., d2,

d3, d4, and N2, N3, N4= Corresponding values for pulleys 2, 3 and 4.

We know that velocity ratio of pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

Similarly, velocity ratio of pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{d_3}{d_4}$$

Multiplying the above equations gives

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \qquad ...(\because N_2 = N_3 \ldots)$$

Slip of Belt

 In the previous articles, we have discussed the motion of belts and shafts assuming a firm frictional grip between the belts and the shafts. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called slip of the belt and is generally expressed as a percentage. The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance.

Let $s_1\%$ = Slip between the driver and the belt, and

$s_2\%$ = Slip between the belt and the follower.

∴ Velocity of the belt passing over the driver per second

$$v = \frac{\pi d_1 . N_1}{60} - \frac{\pi d_1 . N_1}{60} \times \frac{s_1}{100} = \frac{\pi d_1 . N_1}{60}\left(1 - \frac{s_1}{100}\right)$$

and velocity of the belt passing over the follower per second,

$$\frac{\pi d_2 . N_2}{60} = v - v \times \frac{s_2}{100} = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation (i),

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \qquad \dots\left(\text{Neglecting } \frac{s_1 \times s_2}{100 \times 100}\right)$$

$$= \frac{d_1}{d_2}\left(1 - \frac{s_1 + s_2}{100}\right) = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

$$\dots (\text{where } s = s_1 + s_2, \text{ i.e. total percentage of slip})$$

If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

Length of an open Belt Drive

Let

r1 and r2 = Radii of the larger and smaller pulleys, x = Distance between the centres of two pulleys (i.e. O1 O2), and L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. Through

O2, draw O2 M parallel to FE.

From the geometry of the figure, we find that O2 M will be perpendicular to O1 E.

Let the angle MO2 O1 = α radians.

We know that the length of the belt,

L = Arc GJE + EF + Arc FKH + HG

= 2 (Arc JE + EF + Arc FK)

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1 . \alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2 . \alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

From the geometry of the figure, we find that

$$\sin \alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - EM}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

Since α is very small, therefore putting

$$\sin \alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x}$$

Substituting the value of $\alpha = \frac{r_1 - r_2}{x}$

$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x} \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

$$= \frac{\pi}{2}(d. \cdots$$

Length of a Cross Belt Drive
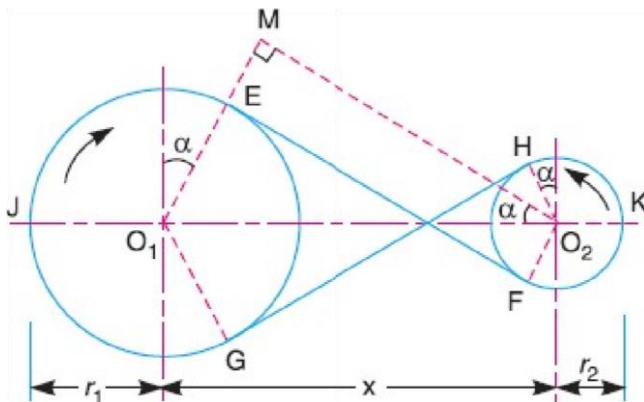
We have already discussed that in a cross belt drive, both the pulleys rotate in opposite directions as shown in Fig.

Let r1 and r2 = Radii of the larger and smaller pulleys, x = Distance between the centres of two pulleys (i.e. O1 O2), and L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H, as shown in Fig. Through O2, draw O2M parallel to FE.

From the geometry of the figure, we find that O2M will be perpendicular to O1E.

Let the angle MO2 O1 = α radians

We know that the length of the belt,

L = Arc GJE + EF + Arc FKH + HG

= 2 (Arc JE + EF + Arc FK)

From the geometry of the figure, we find that

$$\sin \alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + EM}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

Since α is very small, therefore putting

$$\sin \alpha = \alpha \text{ (in radians)} = \frac{r_1 + r_2}{x}$$

$\therefore$ $\qquad$ Arc $JE = r_1 \left(\frac{\pi}{2} + \alpha \right)$

Similarly $\quad$ Arc $FK = r_2 \left(\frac{\pi}{2} + \alpha \right)$

$$EF = MO_2 = \sqrt{(O_1 O_2)^2 - (O_1 M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x \sqrt{1 - \left(\frac{r_1 + r_2}{x} \right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x \left[1 - \frac{1}{2} \left(\frac{r_1 + r_2}{x} \right)^2 + \ldots \right] = x - \frac{(r_1 + r_2)^2}{2x}$$

$$L = 2 \left[r_1 \left(\frac{\pi}{2} + \alpha \right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \left(\frac{\pi}{2} + \alpha \right) \right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1 . \alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \times \frac{\pi}{} \right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + \right.$$

$$= \pi(\cdot$$

Substituting the value of $\alpha = \dfrac{r_1 + r_2}{x}$

$$L = \pi(r_1 + r_2) + \frac{2(r_1 + r_2)}{x} \times (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \qquad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \qquad \text{...(In terms of pulley diameters)}$$

It may be noted that the above expression is a function of $(r1 + r2)$. It is thus obvious that if sum of the radii of the two pulleys be constant, then length of the belt required will also remain constant, provided the distance between centres of the pulleys remain unchanged.

Power transmitted by a Belt

Fig. shows the driving pulley (or driver) A and the driven pulley (or follower) B. We have already discussed that the driving pulley pulls the belt from one side and delivers the same to the other side. It is thus obvious that the tension on the former

side (i.e. tight side) will be greater than the latter side (i.e. slack side) as shown in Fig.

Let T_1 and T_2 = Tensions in the tight and slack side of the belt respectively in newtons,

r_1 and r_2 = Radii of the driver and follower respectively, and

v = Velocity of the belt in m/s.
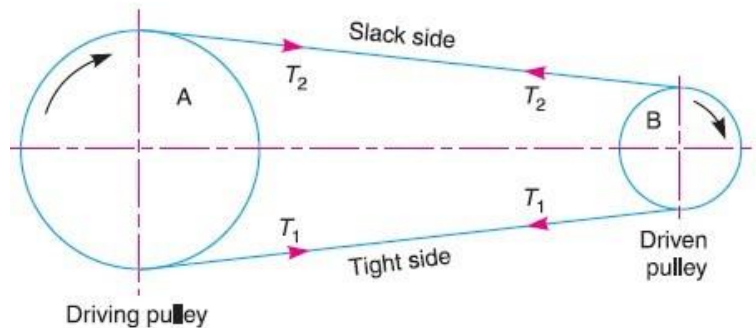
The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (i.e. $T_1 - T_2$).

∴ Work done per second = $(T_1 - T_2) v$ N-m/s

and power transmitted, $P = (T_1 - T_2) v$ W ...(∵ 1 N-m/s = 1 W)

A little consideration will show that the torque exerted on the driving pulley is $(T_1 - T_2) r_1$. Similarly, the torque exerted on the driven pulley i.e. follower is $(T_1 - T_2) r_2$.

Ratio of Driving Tensions for Flat Belt Drive

Consider a driven pulley rotating in the clockwise direction as shown in Fig.
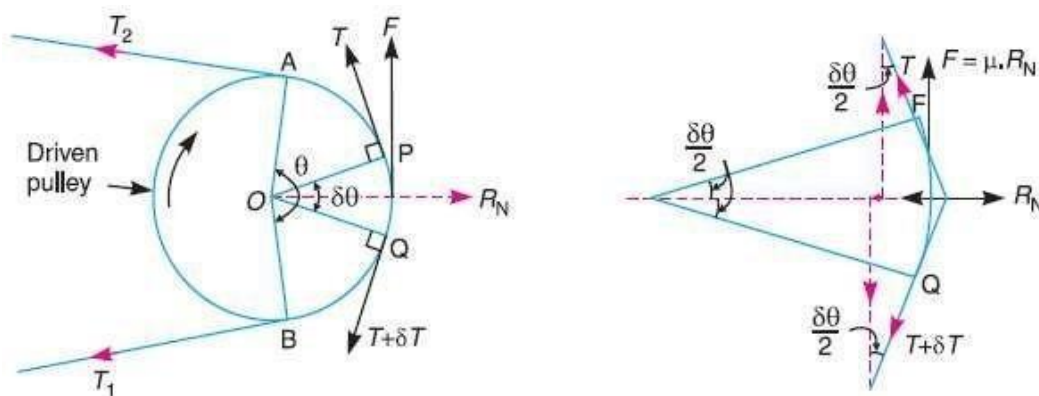
Let T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtend⌐ ⁻
which the belt touches the pulley at ᵉᵗ

Now consider a small portion of the belt *PQ*, su⁻ᵗ
pulley as shown in Fig. 11.15. The belt *PQ* is in eⁿᵐⁱ

1. Tension *T* in the belt at *P*,
2. Tension $(T + \delta T)$ iⁿ ᵗᵗ
3. Normal reᵃᶜᵗ
4. Frⁱ

Resolving all the forces horizontally and equating the same,

$$R_N = (T + \delta T) \sin \frac{\delta\theta}{2} + T \sin\frac{\delta\theta}{2} \qquad \ldots(i)$$

Since the angle $\delta\theta$ is very small, therefore putting $\sin \delta\theta/2 = \delta\theta/2$ in equation (*i*).

$$R_N = (T + \delta T) \frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} = \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2} = T.\delta\theta \qquad \ldots(ii)$$

$$\ldots\left(\text{Neglecting } \frac{\delta T.\delta\theta}{2}\right)$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T) \cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \qquad \ldots(iii)$$

Since the angle $\delta\theta$ is very small, therefore putting $\cos \delta\theta/2 = 1$ in equation (*iii*),

$$\mu \times R_N = T + \delta T - T = \delta T \quad \text{or} \quad R_N = \frac{\delta T}{\mu} \qquad \ldots(iv)$$

Equating the values of R_N from equations (*ii*) and (*iv*),

$$T.\delta\theta = \frac{\delta T}{\mu} \quad \text{or} \quad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 and T_1 and from 0 to θ respectively,

i.e. $$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^\theta \delta\theta \quad \text{or} \quad \log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta \quad \text{or} \quad \frac{T_1}{T_2} = e^{\mu.\theta}$$

Equation (*v*) can be expressed in terms of corresponding logarithm to the base 10, *i.e.*

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

The above expression gives the relation between the tight side and slack side tensions, in terms of coefficient of friction and the angle of contact.

Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called centrifugal tension. At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account. Consider a small portion PQ of the belt subtending an angle d the centre of the pulley as shown in Fig.
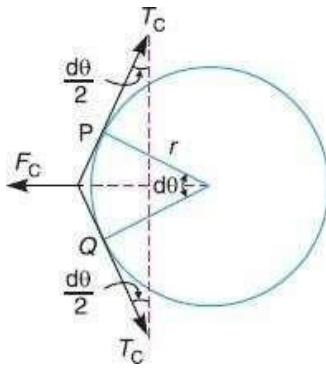
Let m = Mass of the belt per unit length in kg,

v = Linear velocity of the belt in m/s,

r = Radius of the pulley over which the belt runs in metres, and

T_C = Centrifugal tension acting tangentially at P and Q in newtons.

We know that length of the belt PQ

$$= r.\,d\theta$$

and mass of the belt PQ $\qquad = m.\,r.\,d\theta$

∴ Centrifugal force acting on the belt PQ,

$$F_C = (m.r.d\theta)\frac{v^2}{r} = m.d\theta.v^2$$

The centrifugal tension T_C acting tangentially at P and Q keeps the belt in equilibrium.

Now resolving the forces (i.e. centrifugal force and centrifugal tension) horizontally and equating the same, we have

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C = m.d\theta.v^2$$

Since the angle $d\theta$ is very small, therefore, putting $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$, in the above expression,

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2 \quad \text{or} \quad T_C = m \cdot v^2$$

Maximum Tension in the Belt

A little consideration will show that the maximum tension in the belt (T) is equal to the tension in the tight side of the belt (T_{t1}).

Let σ = Maximum safe stress in N/mm²,

b = Width of the belt in mm

t = Thickness of th-

We know that maximum tension in the

T

When centrifugal t-

and wh-

Condition for the Transmission of Maximum Power

We know that power transmitted by a belt,

$$P = (T_1 - T_2) v \qquad \qquad ...(i)$$

where T_1 = Tension in the tight side of the belt in newtons,

                     T_2 = Tension in the slack side of the belt in newtons, and

                     v = Velocity of the belt in m/s.

From Art. 11.14, we have also seen that the ratio of driving tensions is

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu.\theta}} \qquad \qquad ...(ii)$$

Substituting the value of T_2 in equation (i),

$$P = \left(T_1 - \frac{T_1}{e^{\mu.\theta}} \right) v = T_1 \left(1 - \frac{1}{e^{\mu.\theta}} \right) v = T_1 . v . C \qquad \qquad ...(iii)$$

We know that $T_1 = T - T_C$ and for maximum power, $T_C = \dfrac{T}{3}$.

$$T_1 = T - \dfrac{T}{3} = \dfrac{2T}{3}$$

From equation (iv), the velocity of the belt for the maximum power,

$$v = \sqrt{\dfrac{T}{3m}}$$

Gears

Gears are also used for power transmission. This is accomplished by the successive engagement of teeth. The two gears transmit motion by the direct contact like chain drive. Gears also provide positive drive.

The drive between the two gears can be represented by using plain cylinders or discs 1 and 2 having diameters equal to their pitch circles as shown in Figure 3.5. The point of contact of the two pitch surfaces shell have velocity along the common tangent. Because there is no slip, definite motion of gear 1 can be transmitted to gear 2 or vice-versa.

Classify gears

According to the position of axes of the shafts.

 The axes of the two shafts between which the motion is to be transmitted, may be

Parallel, (b) Intersecting, and (c) Non-intersecting and non-parallel.

2. According to the peripheral velocity of the gears. The gears, according to the peripheral velocity of the gears may be classified as :

   (a) Low velocity, (b) Medium velocity, and (c) High velocity

According to the type of gearing.

The gears, according to the type of gearing may be classified as :

External gearing, (b) Internal gearing, and (c) Rack and pinion.

According to position of teeth on the gear surface. The teeth on the gear surface may be

(a) straight, (b) inclined, and (c) curved

NOMENCLATURE OF GEAR TEETH
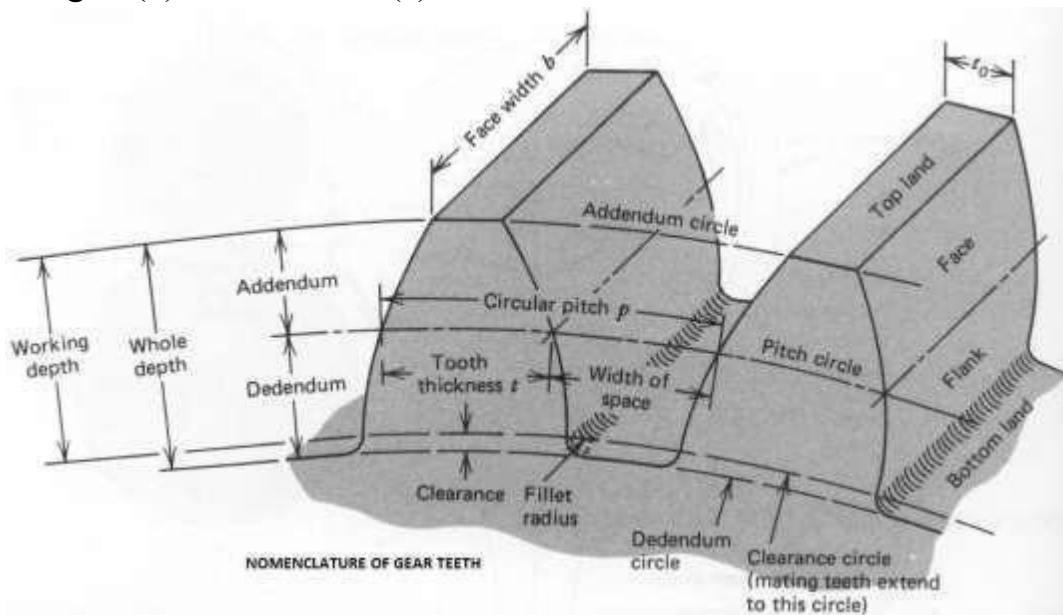
Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. Pitch circle diameter. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as pitch diameter.

3. Pitch point. It is a common point of contact between two pitch circles. 4. Pitch surface. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point.

6. Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called root circle.

10.   Circular pitch. It is the distance measured on the circumference of the pitchcircle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by pc.

Diametral pitch. It is the ratio of number of teeth to the pitch circle diameter in millimetres.

It is denoted by pd

. Mathematically, Diametral pitch,$Pd=T/D$

, D=diameter of pitch circle

T=number of teeth on the wheel

Module. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. Mathematically, $m=D/T$

Working depth. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16.   Tooth thickness. It is the width of the tooth measured along the pitch circle.

17.   Tooth space . It is the width of space between the two adjacent teeth measured along the pitch circle.

18.   Backlash. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19.   Face of tooth. It is the surface of the gear tooth above the pitch surface. 20. Flank of tooth. It is the surface of the gear tooth below the pitch surface.

21. Top land. It is the surface of the top of the tooth.

22. Face width. It is the width of the gear tooth measured parallel to its axis.

23. Profile. It is the curve formed by the face and flank of the tooth.

24. Fillet radius. It is the radius that connects the root circle to the profile of the tooth.

25. Path of contact. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. *Length of the path of contact. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. ** Arc of contact. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, i.e.

(a) Arc of approach. It is the portion of the path of contact from the beginning of the engagement to the pitch point

Simple gear train
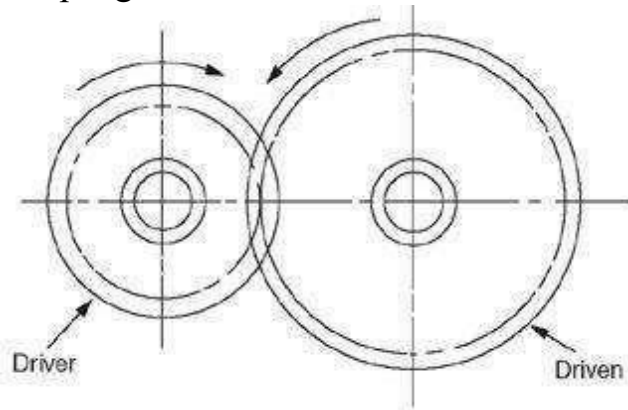
Driver   Driven

A simple gear train uses two gears, which may be of different sizes. If one of these gears is attached to a motor or a crank then it is called the driver gear. The gear that is turned by the driver gear is called the driven gear. The input and the output shaft are necessarily being parallel to each other. In this gear train, there are series of gears which are capable of receiving and transmitting motion from one gear to another. They may mesh externally or internally. Each gear rotates about separate axis fixed to the frame. Two gears may be external meshing and internal meshing.

Velocity ratio:

N1/N2=T2/T1=d2/d1

Compound gear train
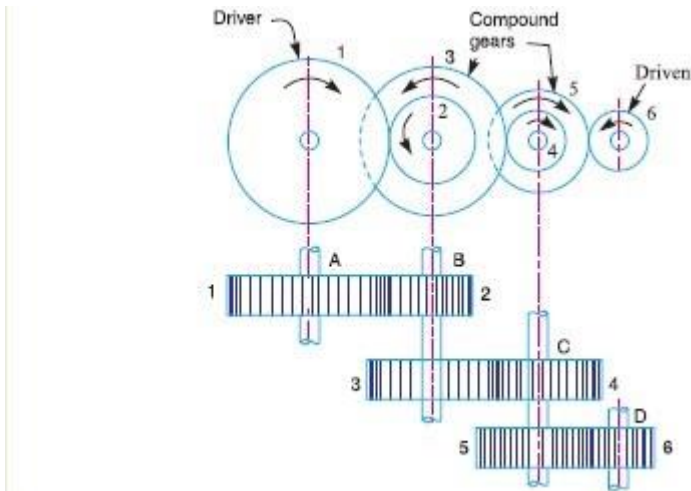
When there are more than one gear on a shaft, , it is called a compound train of gear.

in a simple train of gears do not effect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven

In a compound train of gears, as shown in Fig .the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also

compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let N_1 = Speed of driving gear 1,
T_1 = Number of teeth on driving gear 1,
$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and
$T_2, T_3, ..., T_6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \text{...(i)}$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \qquad \text{...(ii)}$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \qquad \text{...(iii)}$$

The speed ratio of compound gear train is obtained by multiplying the equations (i), (ii) and (iii),

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad \frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

Epicyclic gear train

in an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig., where a gear A and thearm C have a common axis at O 1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O2, about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or vice- versa, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O 1), then the gear B is forced to rotate upon and around gear A. Such a motion is called epicyclic and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as epicyclic gear trains (epi. means upon and cyclic means around). The epicyclic gear trains may be simple or compound. The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.
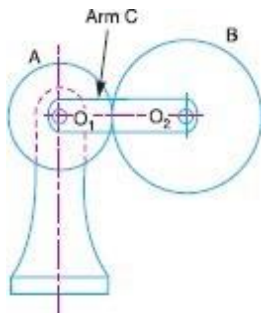
UNIT 4:GOVERNORS & FLYWHEELS

Introduction

The governor is a device which is regulate the mean speed of an engine, when there are variations in the load, during long periods. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required.the governor has no influence over cyclic speed fluctuation.
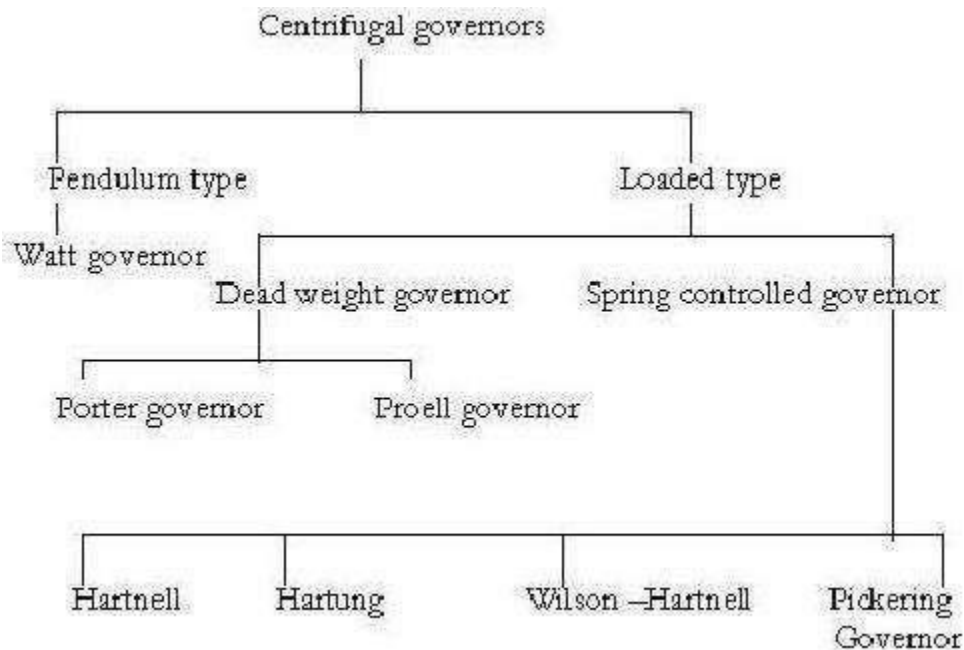
Types of governor

Governors are classified based upon two different principles. These are:Centrifugal governors are further classified as –
• 	• Centrifugal governor

- • Inertia governors

Centrifugal governors are further classified as –

```
                    Centrifugal governors
                            |
        ┌───────────────────┴───────────────────┐
   Pendulum type                           Loaded type
        |                   ┌───────────────────┴───────────┐
   Watt governor      Dead weight governor      Spring controlled governor
                   ┌────────┴────────┐                      |
             Porter governor   Proell governor              |
                                                            |
                         ┌──────────┬──────────────┬────────┴────────┐
                     Hartnell    Hartung     Wilson –Hartnell    Pickering
                                                                 Governor
```

Watt Governor

The simplest form of a centrifugal governor is a Watt governor, It consist of pair of two balls and which is attached with the spindle with help of arms. The upper arm pinned at point O. the lower arm are fixed connect to the sleeve. The sleeve freely move on the spindle which is driven by engine. The spindle rotate the balls take of position depending upon speed of spindle

The arms of the governor may be connected to the spindle in the following three ways :

1. The pivot P, may be on the spindle axis.

2. The pivot P, may be offset from the spindle axis and the arms when produced intersect at O.

3. The pivot P, may be offset, but the arms cross the axis at O.

Let
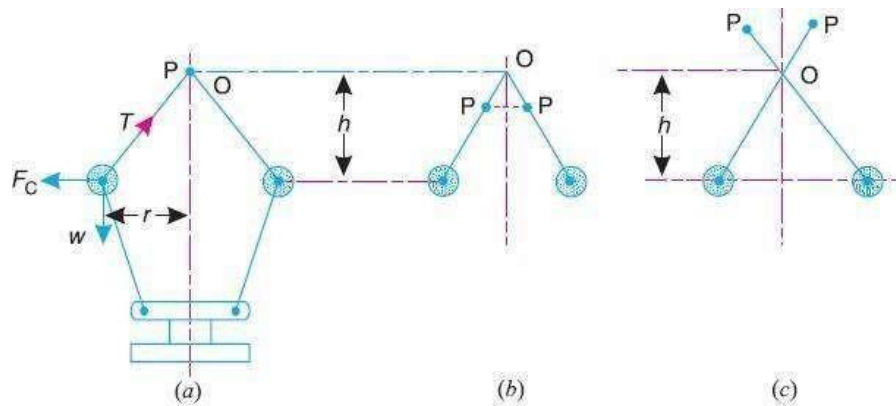
Watt governor.

m = Mass of the ball in kg,

w = Weight of the ball in newtons = m.g, T = Tension in the arm in newtons, ω= Angular velocity of the arm and ball about the spindle axis in rad/s, r = Radius of the path of rotation of the ball i.e. horizontal distance from the centre of the ball to the spindle axis in metres,

FC = Centrifugal force acting on the ball in newtons = mv2/r

h = Height of the governor in metres.

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls. Now, the ball is in equilibrium under the action of

1. the centrifugal force (FC) acting on the ball,

2. the tension (T) in the arm,

3. the weight(w) of the ball.

Taking moments about point O, we have

$$F_c \times h = w \times r = m.g.r$$

or $\qquad m.\omega^2 r.h = m.g.r \qquad$ or $\qquad h = g/\omega^2 \qquad \qquad \ldots(i)$

When g is expressed in m/s^2 and ω in rad/s, then h is in metres. If N is the speed in r.p.m., then

$$\omega = 2\pi N/60$$

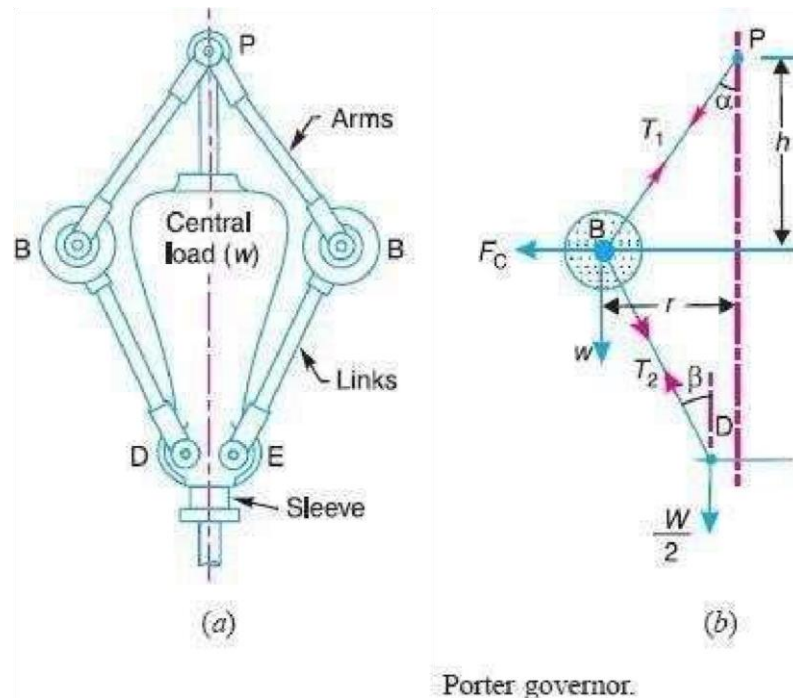$$\therefore \quad h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ metres} \qquad \ldots(\because g = 9.81 \text{ m/s}^2)\ldots(ii)$$

Porter Governor

It differs from the watt governor is in the use of a heavily weighted sleeve. The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.

Consider the forces acting on one-half of the governor

Porter governor.

Let m = Mass of each ball in kg,

w = Weight of each ball in newtons = m.g, M = Mass of the central load in kg, W

= Weight of the central load in newtons = M.g, r = Radius of rotation in metres, h

= Height of governor in metres , N = Speed of the balls in r.p.m .,

ω= Angular speed of the balls in rad/s= $2\pi N/60$ rad/s,

FC = Centrifugal force acting on the ballin newtons = mv^2/r, T1 = Force in the arm in newtons,
T2 = Force in the link in newtons,

α= Angle of inclination of the arm (or upper link) to the vertical, and

β= Angle of inclination of the link (or lower link) to the vertical.

The weight of arms and weight of suspension links and effect of friction to the movement of sleeve are neglected.

Though there are several ways of determining the relation between the height of the governor (h) and the angular speed of the balls (ω).

1. Method of resolution of forces

Considering the equilibrium of the forces acting at D, we have

$$T_2 \cos \beta = \frac{W}{2} = \frac{M \cdot g}{2}$$

or

$$T_2 = \frac{M \cdot g}{2 \cos \beta}$$

Again, considering the equilibrium of the forces acting on B. The point B is in equilibrium under the action of the following forces, as shown in Fig. 18.3 (b).

(i) The weight of ball ($w = m.g$),
(ii) The centrifugal force (F_c),
(iii) The tension in the arm (T_1), and
(iv) The tension in the link (T_2).

Resolving the forces vertically,

$$T_1 \cos \alpha = T_2 \cos \beta + w = \frac{M \cdot g}{2} + m.g \qquad \ldots (ii)$$

$$\ldots \left(\because T_2 \cos \beta = \frac{M \cdot g}{2} \right)$$

Resolving the forces horizontally,

$$T_1 \sin \alpha + T_2 \sin \beta = F_c$$

$$T_1 \sin \alpha + \frac{M \cdot g}{2 \cos \beta} \times \sin \beta = F_c \qquad \ldots \left(\because T_2 = \frac{M \cdot g}{2 \cos \beta} \right)$$

$$T_1 \sin \alpha + \frac{M \cdot g}{2} \times \tan \beta = F_c$$

$$\therefore \quad T_1 \sin \alpha = F_c - \frac{M \cdot g}{2} \times \tan \beta \qquad \ldots (iii)$$

Proell Governor

The proell governor has ball fixed at B and C at extension of link DF and EG. The aem FP and GQ are pivoted at P and Q respectvily

Consider the equilibrium forces one-half of governor as shown in fig b.the instantneous centr I lies on on the intersection of line PF produced and from D drawn perpendicular to spindle axis. The perpendicular BM is drawn on ID
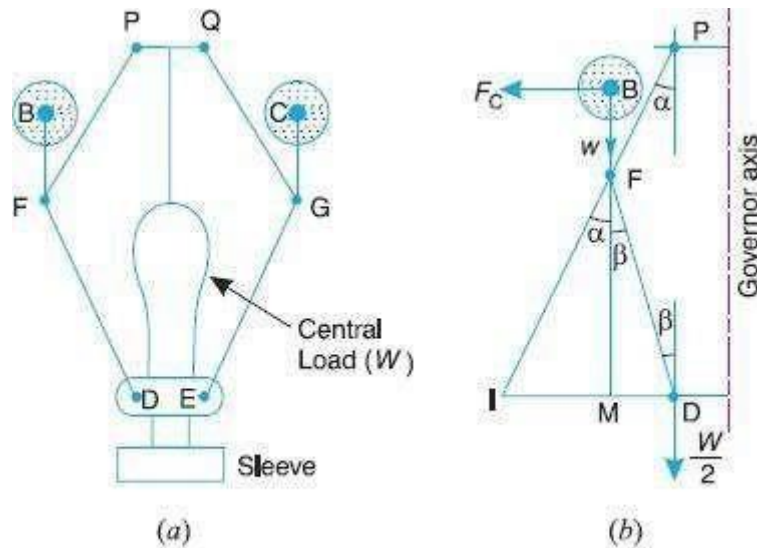
Fig. Proell governor.

Taking moments about I, using the same notations

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \qquad \ldots(i)$$

$$\therefore \qquad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \qquad \ldots(\because ID = IM + MD)$$

Multiplying and dividing by FM, we have

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

We know that $F_C = m.\omega^2 r$; $\tan\alpha = \dfrac{r}{h}$ and $q = \dfrac{\tan\beta}{\tan\alpha}$

$$\therefore \qquad m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

and

$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \dfrac{M}{2}(1+q)}{m}\right]\frac{g}{h} \qquad \ldots(ii)$$

Substituting $\omega = 2\pi N/60$, and $g = 9.81 \text{ m/s}^2$, we get

$$N^2 = \frac{FM}{BM}\left[\frac{m + \dfrac{M}{2}(1+q)}{m}\right]\frac{895}{h} \qquad \ldots(iii)$$

Hartnell Governor

A Hartnell governor is a spring loaded governor .It consists of two bell crank levers pivoted at the points O,O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.

Let m = Mass of each ball in kg,

M = Mass of sleeve in kg,

r_1 = Minimum radius of rotation in metres, r_2

= Maximum radius of rotation in metres

ω_1 = Angular speed of the governor at minimum radius in rad/s,

ω_2 = Angular speed of the governor at maximum radius in rad/s,

S1 = Spring force exerted on the sleeve S2 = Spring force exerted on the sleeve at FC1 = Centrifugal force = $m (\omega_1)2 r$ FC2 = Centrifugal force at = m (ω_2)2 r2,

s = Stiffness of the spring or the force required to compress the spring by one mm, x = Length of the vertical or ball arm of the lever in metres, y = Length of the horizontal or sleeve arm of the lever in metres, and r = Distance of fulcrum O from the governor axis or the radius of rotation when the governor is in mid-position.

Sensitiveness of Governors

A governor is said to be sensitive, if its change of speed s from no Load to full load may

be as small a fraction of the mean equilibrium speed as possible and the corresponding sleeve lift may be as large as possible.
Suppose ω1 = max. Equilibrium speed ω2 = min. equilibrium speed

ω = mean equilibrium speed = $(\omega_1 + \omega_2)/2$ Therefore sensitiveness = $(\omega_1 - \omega_2)/2$
Stability of Governors

A governor is said to be stable when for every speed within the working range there is definite configuration i.e. there is only one radius of rotation of the governor balls at which the governor equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

Isochronous Governors

This is an extreme case of sensitiveness. When the equilibrium speed is constant for all radii of rotation of the balls within the working range, the governor is said to be in isochronism. This means that the difference between the maximum and minimum equilibrium speeds is zero and the sensitiveness shall be infinite.

FLYWHEEL:-

A flywheel is a wheel of heavy mass mounted on the crank shaft and it stores energy during the period when the supply of energy is more

during the period when the fly wheel absorbs energy its speed increases and during the period when it releases energy its speed decreases.

In engine, the flywheel absorbs the stroke and gives out the energy during idle strokes and thus keeps the maximum speed and minimum speed of crankshaft near the mean shaft in a thermodynamic cycle. In power press, the flywheel absorbs the mechanical energy produced by electric motor during idle period and gives the energy when actual operation is performed. In this way with the use of flywheel, motor of smaller capacity is able to serve the purpose.

Fluctuation of Energy

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider the turning moment diagram for a single cylinder double acting steam engine as shown in Fig.

We see that the mean resisting torque line AF cuts the turning moment diagram at points B, C, D and E. When the crank moves from a to p, the work done by the engine is equal to the area aBp, whereas the energy required is represented by the area aABp. In other words, the engine has done less work (equal to the area a AB) than the requirement. This amount of energy is taken from the flywheel and hence

the speed of the flywheel decreases. Now the crank moves from p to q, the work done by the engine is equal to the area pBbCq, whereas the requirement of energy is represented by the area pBCq. Therefore, the engine has done more work than the requirement.
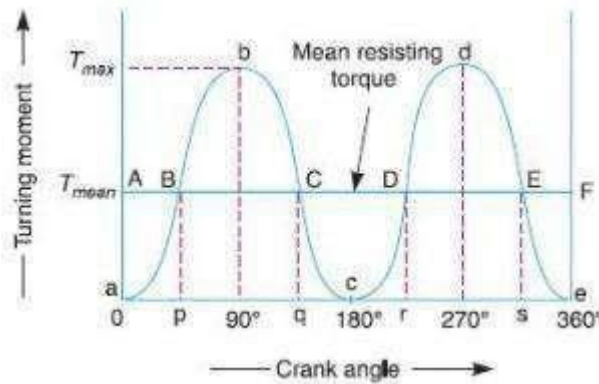
This excess work (equal to the area BbC) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from p to q. Similarly, when the crank moves from q to r, more work is taken from the engine than is developed. This loss of work is represented by the area C c D. To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from q to r. As the crank moves from r to s, excess energy is again developed given by the area D d E and the speed again increases. As the piston moves from s to e, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called fluctuations of energy.

UNIT 5: Balancing of machine: Concept of static and dynamic balancing

Explain the concept of balancing:

Balancing is the process of eliminating or at least reducing the ground forces and/or moments. It is achieved by changing the location of the mass centres of links. Balancing of rotating parts is a well known problem. A rotating body with fixed rotation axis can be fully balanced i.e. all the inertia forces and moments. For mechanism containing links rotating about axis which are not fixed, force balancing

is possible, moment balancing by itself may be possible, but both not possible. We generally try to do force balancing. A fully force balance is possible, but any action in force balancing severe the moment balancing

Balancing of rotating masses: The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called balancing of rotating masses.

Static balancing: The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of the rotation. This is the condition for static balancing.

Dynamic balancing: The net couple due to dynamic forces acting on the shaft is equal to zero. The algebraic sum of the moments about any point in the plane must be zero.
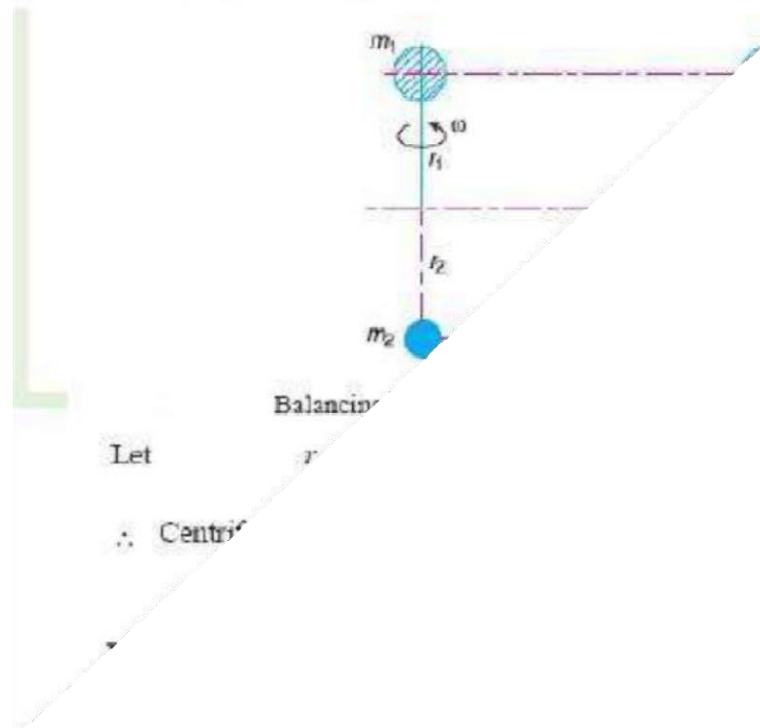

Static balancing of rotating mass

Balancing of a single rotating mass by single mass rotating in the same plane:

Consider a disturbing mass m_1 attached to a shaft rotating at ω rad/s as shown in Fig. Let r_1 be the radius of rotation of the mass m_1 (i.e. distance between the axis of rotation of the s' and the centre of gravity of the mass m_1).

We know that the centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

This centrifugal force acts radially outwards and thus produces ber shaft. In order to counteract the effect of this force, a balancing mass (m_2) same plane of rotation as that of disturbing mass (m_1) such that the c two masses are equal and opposite.

Balancin

Let r

∴ Centri

CASE 2:

BALANCING OF A SINGLE ROTATING MASS BY TWO MASSES

ROTATING IN DIFFERENT PLANES.

There are two possibilities while attaching two balancing masses:

1.   The plane of the disturbing mass may be in between the planes of the two balancing masses.

2.   The plane of the disturbing mass may be on the left or right side of two planes containing the balancing masses.

which are parallel to the plane of rotation of the disturbing mass i) the net dynamic force

acting on the shaft must be equal to zero, i.e. the centre of the masses of the system must lie on the axis of rotation and this is the condition for static balancing ii) the net couple due to the dynamic forces acting on the shaft must be equal to zero, i.e. the algebraic sum of the moments about any point in the plane
must be zero. The conditions i) and ii) together give dynamic balancing. THE PLANE OF THE DISTURBING MASS LIES IN BETWEEN THE PLANES

OF THE TWO BALANCING MASSES.

Consider the disturbing mass m lying in a plane A which is to be balanced by two rotating masses m_1 and m_2 lying in two different planes M and N which are parallel to the plane A as shown.

Let r, r_1 and r_2 be the radii of rotation of the masses in planes A, M and N respectively. Let L_1, L_2 and L be the distance between A and M, A and N, and M and N respectively.
Now,
The centrifugal force exerted by the mass m in plane A will be,

$$F_c = m\,\omega^2 r \quad\text{------------------(1)}$$

Similarly,
The centrifugal force exerted by the mass m_1 in plane M will be,

$$F_{c1} = m_1\,\omega^2 r_1 \quad\text{------------------(2)}$$

And the centrifugal force exerted by the mass m_2 in plane N will be,

$$F_{c2} = m_2\,\omega^2 r_2 \quad\text{-----------------}$$

For the condition of static balancing,

$$F_c = F_{c1} + F_{c2}$$

or

Now, to determine the magnitude of balancing force in the plane 'M' or the dynamic force at the bearing 'O' of a shaft, take moments about ' P ' which is the point of intersection of the plane N and the axis of rotation.

Therefore,

$$F_{c1} \times L = F_c \times L_2$$
$$\text{or } m_1 \omega^2 r_1 \times L = m\omega^2 r \times L_2$$
Therefore,
$$m_1 r_1 L = mrL_2 \quad \text{or} \quad m_1 r_1 = mr\frac{L_2}{L} \text{---------(5)}$$

Similarly, in order to find the balancing force in plane 'N' or the dynamic force at the bearing 'P' of a shaft, take moments about ' O ' which is the point of intersection of the plane M and the axis of rotation.

Therefore,

$$F_{c2} \times L = F_c \times L_1$$
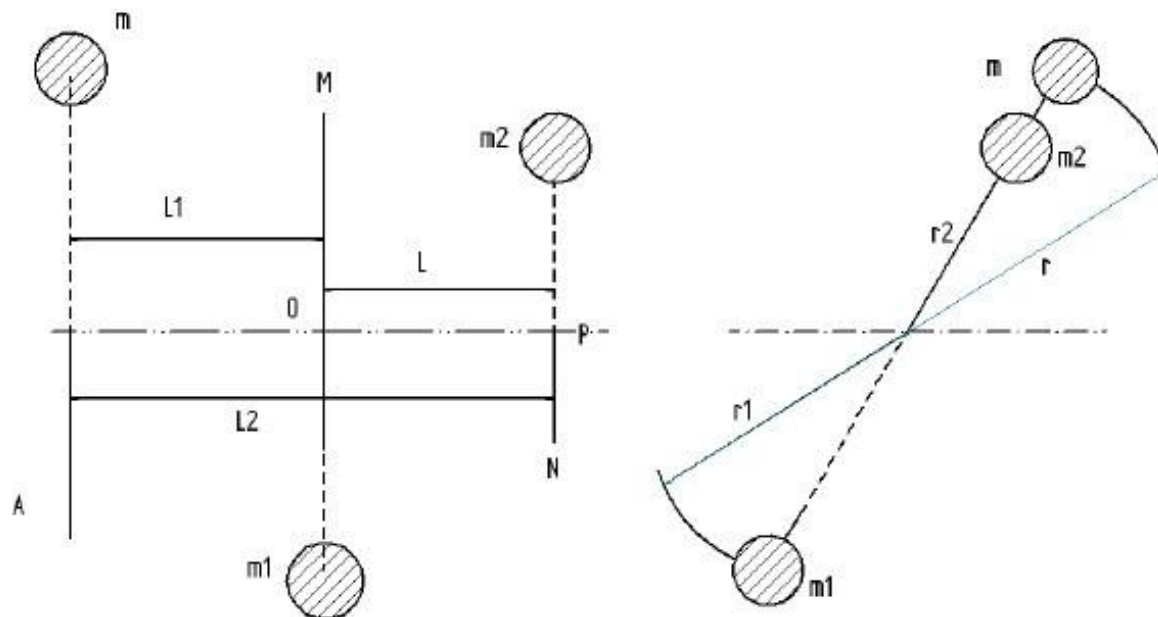$$\text{or } m_2 \omega^2 r_2 \times L = m\omega^2 r \times L_1$$
Therefore,
$$m_2 r_2 L = mrL_1 \quad \text{or} \quad m_2 r_2 = mr\frac{L_1}{L} \text{---------(6)}$$

For dynamic balancing equations (5) or (6) must be satisfied along with equation (4).

CASE 2(II):

WHEN THE PLANE OF THE DISTURBING MASS LIES ON ONE END OF THE TWO PLANES CONTAINING THE BALANCING MASSES.

When the plane of the disturbing mass lies on one end of the planes of the balancing masses

$F_{c1} \times L = F_c \times L_2$

or $m_1 \omega^2 r_1 \times L = m \omega^2 r \times L_2$

Therefore,

$m_1 r_1 L = mrL_2$ or $m_1 r_1 = mr\dfrac{L_2}{L}$ - - - -

Similarly, to find the balancing force in the

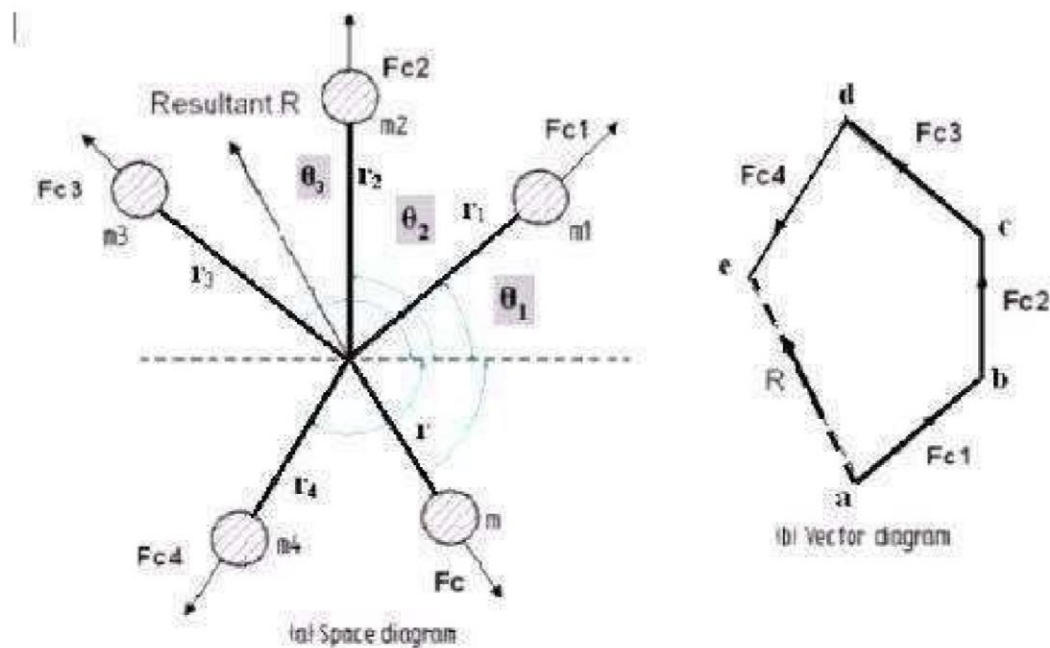$F_{c2} \times L = F_c \times L_1$

or $m_2 \omega^2 r$

The

CASE 3:
BALANCING OF SEVERAL MASSES ROTATING IN THE SAME PLANE

(a) Space diagram   (b) Vector diagram

BALANCING OF SEVERAL MASSES ROTATING IN THE SAME PLANE

Consider a rigid rotor revolving with a constant angular velocity _ rad/s. A number of masses say, four are depicted by point masses at different radii in the same transverse plane

If m_1, m_2, m_3 and m_4 are the masses revolving at radii r_1, r_2, r_3 and r_4 respectively in th...
same plane.
The centrifugal forces exerted by each of the masses are F_{c1}, F_{c2}, F_{c3} and F_{c4} resp...
Let F be the vector sum of these forces. i.e.

$$F = F_{c1} + F_{c2} + F_{c3} + F_{c4}$$
$$= m_1 \omega^2 r_1 + m_2 \omega^2 r_2 + m_3 \omega^2 r_3 + m_4 \omega^2 r_4 -$$

The rotor is said to be statically balanced if the vect...
is not zero, i.e. the rotor is unbalanced, then intr...
of mass 'm' at radius 'r' to balance the rotor...

$$m_1 \omega^2 r_1 + m_2 \omega^2 r_2 + m_3 \omega^{}$$

$$m_1 r_1 + m_2 r_2 + $$
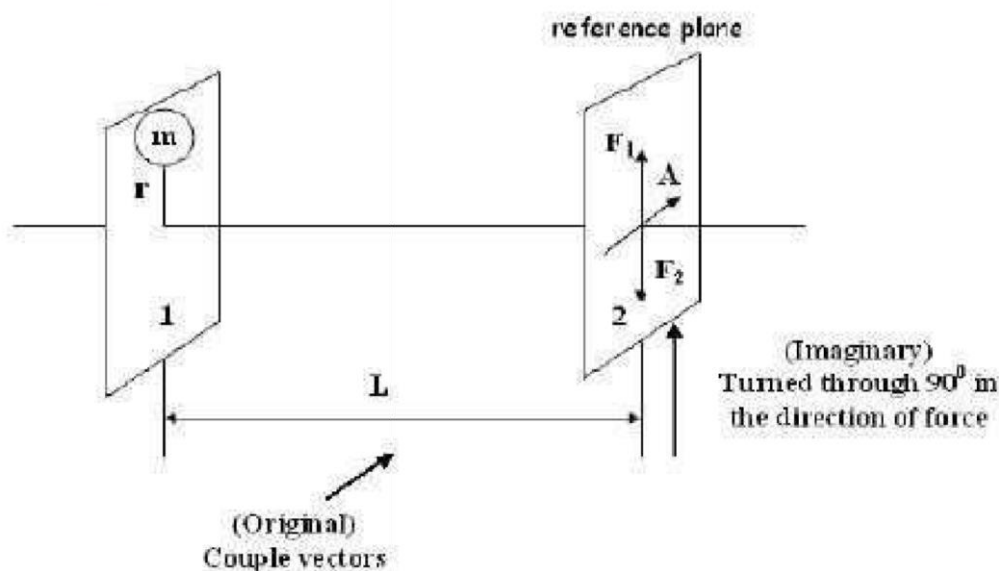
The magnitude o...
In general, i...

CASE 4:

BALANCING OF SEVERAL MASSES ROTATING IN DIFFERENT PLANES

When several masses revolve in different planes, they may be transferred to a reference plane and this reference plane is a plane passing through a point on the axis of rotation and perpendicular to it.

When a revolving mass in one plane is transferred to a reference plane, its effect is to cause a force of same magnitude to the centrifugal force of the revolving mass to act in the reference plane along with a couple of magnitude equal to the product of the force and the distance between the two planes.

In order to have a complete balance of the several revolving masses in different planes,

1.	the forces in the reference plane must balance, i.e., the resultant force must be zero and

2.	the couples about the reference plane must balance i.e., the resultant couple must be zero.

A mass placed in the reference plane may satisfy the first condition but the couple balance is satisfied only by two forces of equal magnitude in different planes. Thus, in general, two planes are needed to balance a system of rotating masses balanicing of reciprocating engine

UNIT 6:VIBRATION OF MACHINE PARTS

Introduction

When elastic bodies such as a spring, a beam and a shaft are displaced from the equilibrium position by the application of external forces, and then released, they execute a vibratory motion.

Classify vibrations

Longitudinal vibrations. When the particles of the shaft or disc moves parallel to the axis of the shaft, as shown in Fig (a), then the vibrations are known as longitudinal vibrations.

When no external force acts on the body, after giving it an initial displacement, then the body is said to be under free or natural vibrations. The frequency of the free vibrations is called free or natural frequency.

2.	Transverse vibrations. When the particles of the shaft or disc move approximately perpendicular to the axis of the shaft

3.   Torsional vibrations. When the particles of the shaft or disc move in a circle about the axis of the shaft, as shown in Fig.(c), then the vibrations are known as torsional vibrations. In this case, the shaft is twisted and untwisted alternately and the torsional shear stresses are inducedin the shaft.

2- Forced vibrations.

When the body vibrates under the influence of external force, then the body is said to be under forced vibrations. The external force applied to the body is a periodic disturbing force created by unbalance. The vibrations have the same frequency as the applied force.

3- Damped vibrations.

When there is a reduction in amplitude over every cycle of vibration, the motion is said to be damped vibration. This is due to the fact that a certain amount of energy possessed by the vibrating system is always dissipated in overcoming frictional resistances to the motion.

Define with respect to vibration Cycle:

Amplitude:

Time Period:

1. Period of vibration or time period. It is the time interval after which the motion is repeated itself. The period of vibration is usually expressed in seconds.

2. Cycle. It is the motion completed during one time period.

3. Frequency. It is the number of cycles described in one second. In S.I. units, the frequency is expressed in hertz (briefly written as Hz) which is equal to one cycle per second.

State the causes of Vibration

Unbalance: This is basically in reference to the rotating bodies. The uneven distribution of mass in a rotating body contributes to the unbalance. A good example of unbalance related vibra on would be the ─vibra ng alert in our mobile phones. Here a small amount of unbalanced weight is rotated by a motor causing

the vibration which makes the mobile phone to vibrate. You would have experienced the same sort of vibration occurring in your front loaded washing machines that tend to vibrate during the —spinning mode.

Misalignment: This is an other major cause of vibration particularly in machines that are driven by motors or any other prime movers.

Bent Shaft: A rotating shaft that is bent also produces the the vibrating effect since it losses it rotation capability about its center.

Gears in the machine: The gears in the machine always tend to produce vibration, mainly due to their meshing. Though this may be controlled to some extent, any problem in the gearbox tends to get enhanced with ease.

Bearings: Last but not the least, here is a major contributor for vibration. In majority of the cases every initial problem starts in the bearings and propagates to the rest of the members of the machine. A bearing devoid of lubrication tends to wear out fast and fails

quickly, but before this is noticed it damages the remaining components in the machine and an initial look would seem as if something had gone wrong with the other components leading to the bearing failure.

Effects of vibration:

(a)   Bad Effects: The presence of vibration in any mechanical system produces unwanted noise, high stresses, poor reliability, wear and premature failure of parts. Vibrations are a great source of human discomfort in the form of physical and mental strains.

(b)   Good Effects: A vibration does useful work in musical instruments, vibrating screens, shakers, relive pain in physiotherapy • -unbalance is its main cause, so balancing of parts is necessary.
• using shock absorbers.

• using dynamic vibration absorbers.

• providing the screens (if noise is to be reduced)