UTKAL INSTITUTE OF ENGINEERING AND TECHNOLOGY

LECTURE NOTES
ON
STRUCTURAL MECHANICS
DIPLOMA, CIVIL ENGINEERING, 3RD SEMESTER

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COURSE CONTENTS

1. Review Of Basic Concepts

1.1 Basic Principle of Mechanics: Force, Moment, support conditions, Conditions of equilibrium, C.G & MI, Free body diagram **1.2** Review of CG and MI of different sections

2. Simple And Complex Stress, Strain

2.1 Simple Stresses and Strains

Introduction to stresses and strains: Mechanical properties of materials – Rigidity, Elasticity, Plasticity, Compressibility, Hardness, Toughness, Stiffness, Brittleness, Ductility, Malleability, Creep, Fatigue, Tenacity, Durability, Types of stresses -Tensile, Compressive and Shear stresses, Types of strains - Tensile, Compressive and Shear strains, Complimentary shear stress - Diagonal tensile / compressive Stresses due to shear, Elongation and Contraction, Longitudinal and Lateral strains, Poisson's Ratio, Volumetric strain, computation of stress, strain, Poisson's ratio, change in dimensions and volume etc, Hooke's law - Elastic Constants, Derivation of relationship between the elastic constants.

2.2 Application of simple stress and strain in engineering field:

Behaviour of ductile and brittle materials under direct loads, Stress Strain curve of a ductile material, Limit of proportionality, Elastic limit, Yield stress, Ultimate stress, Breaking stress, Percentage elongation, Percentage reduction in area, Significance of percentage elongation and reduction in area of cross section, Deformation of prismatic bars due to uniaxial load, Deformation of prismatic bars due to its self weight.

2.3 Complex stress and strain

Principal stresses and strains: Occurrence of normal and tangential stresses, Concept of Principal stress and Principal Planes, major and minor principal stresses and their orientations, Mohr's Circle and its application to solve problems of complex stresses

3. Stresses In Beams and Shafts

3.1 Stresses in beams due to bending: Bending stress in beams – Theory of simple bending – Assumptions – Moment of resistance – Equation for Flexure– Flexural stress distribution – Curvature of beam – Position of N.A. and Centroidal Axis – Flexural rigidity – Significance of Section modulus

3.2 Shear stresses in beams: Shear stress distribution in beams of rectangular, circular and standard sections symmetrical about vertical axis.

3.3 Stresses in shafts due to torsion: Concept of torsion, basic assumptions of pure torsion, torsion of solid and hollow circular sections, polar moment of inertia, torsional shearing stresses, angle of twist, torsional rigidity, equation of torsion

3.4 Combined bending and direct stresses: Combination of stresses, Combined direct and bending stresses, Maximum and Minimum stresses in Sections, Conditions for no tension, Limit of eccentricity, Middle third/fourth rule, Core or Kern for square, rectangular and circular sections, chimneys, dams and retaining walls

4. Columns and Struts

4.1 Columns and Struts, Definition, Short and Long columns, End conditions, Equivalent length / Effective length, Slenderness ratio, Axially loaded short and long column, Euler's theory of long columns, Critical load for Columns with different end conditions **5. Shear Force and Bending Moment**

5.1 Types of loads and beams:
Types of Loads: Concentrated (or) Point load, Uniformly Distributed load (UDL), Types of Supports: Simple support, Roller support, Hinged support, Fixed support, Types of Reactions: Vertical reaction, Horizontal reaction, Moment reaction, Types of Beams based on support conditions: Calculation of support reactions using equations of static equilibrium.
5.2 Shear force and bending moment in beams:
Shear Force and Bending Moment: Signs Convention for S.F. and B.M, S.F and B.M of general cases of determinate beams with concentrated loads and udl only, S.F and B.M diagrams for Cantilevers, Simply supported beams and Over hanging beams, Position of maximum BM, Point of contra flexure, Relation between intensity of load, S.F and B.M.
6. Slope and Deflection
6.1 Introduction: Shape and nature of elastic curve (deflection curve); Relationship between slope, deflection and curvature (No derivation), Importance of slope and deflection.
6.2 Slope and deflection of cantilever and simply supported beams under concentrated and uniformly distributed load (by Double Integration method, Macaulay's method).
7. Indeterminate Beams
7.1 Indeterminacy in beams, Principle of consistent deformation/compatibility, Analysis of propped cantilever, fixed and two span continuous beams by principle of superposition, SF and BM diagrams (point load and udl covering full span)
8. Trusses
8.1 Introduction: Types of trusses, statically determinate and indeterminate trusses, degree of indeterminacy, stable and unstable trusses, advantages of trusses.
8.2 Analysis of trusses: Analytical method (Method of joints, method of Section)

Sl. No	Name of Authors	Titles of Book	Name of Publisher
1	R.Subramanian	Strength of Materials	Oxford Publication
2	S.Rammrutham,	Theory of structure	Dhanpat Rai Publications
3	V.N.Vazirani&M.M. Rathwani	Analysis of Structures-Vol.I&II	Khanna Publication

This interaction can occur in the following

1. When there is direct contact between
   Ex- A person pushing a box with
2. when the bodies are physically Sepa
   Ex- Gravitational, electrical and Mag

- System of forces can be broadly divid
  (a) coplaner force System.
  (b) Non- Coplaner force System.

Coplanar force System:

System of forces acting in a Sing
body is defined as coplanar force syst

Non-Coplanar force System:

System of forces acting in differen
body is defined as non- coplanar

- Catagories of force System:

(a) Concurrent force System:—

Set of forces Converging or dive
point on the body is defined as Concurr
System.

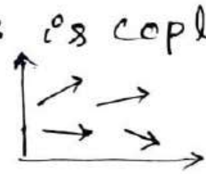
(i) If the Set of concurrent forces is a
plane then it is defined as Coplanar C
force System.

(ii) If the Set of concurrent forces is ac
different planes the it is defined as n
Concurrent force System.

Force System.

Similarly, Non-concurrent force system c
or Non-coplanar in nature.

Ex- A set of parallel forces is coplanar non
force System.



(c) Collinear Force System:-

- Set of force acting along a straight line is a Collinear force System. Collinear force falls under the catagory of coplanar force system.
- A single force which produces the same a number of forces acting together is resultant of these forces.
- If the forces are acting in a straight lin resultant is equal to the algebraic sum forces.
- If the forces are acting in different directi resultant is obtained by;

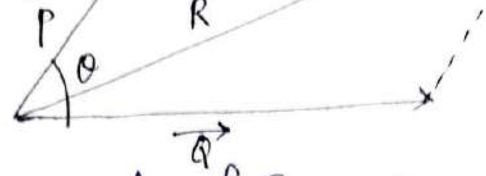
    (a) Law of parallelogram of forces.

    (b) Law of triangle of force

    (c) Law of polygon of force

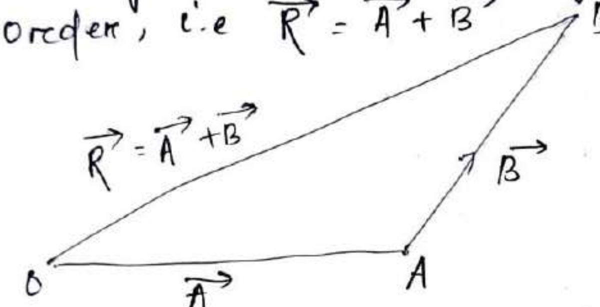
(a) Law of parallelogram of forces:-

It states that if two forces at a point of a body, be reprented in ma direction by the two adjacent sides of a then their resultant is represented in m



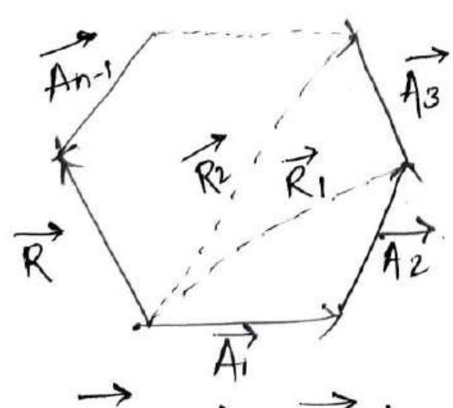
(b) Law of triangle of forces :-

If two non-zero vectors are represent
Sides of a triangle taken in the same order
resultant is given by the closing side o
opposite order, i.e $\vec{R} = \vec{A} + \vec{B}$



(c) Law of polygon of forces :-

It states that if many forces act
are represented in magnitude and directi
Sides of a polygon taken in order, then
is given by the closing side of the polygo
opposite order.



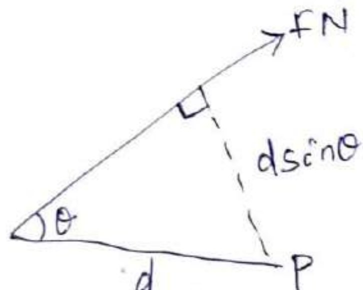
$\vec{R} = \vec{A_1} + \vec{A_2} + \cdots \vec{A}_{n-1}$

... of a force about a point is;

Magnitude × perpendicular dista[nce]
line of action of the
point.

- When there are several forces acting o[n] ... moments about a point can be added s[o] ... positive direction (clockwise or anticlockwi[se]) and is considered for each moment.

find the moment of F and P.



find the moment of F about P in the
find the moment of F about P ~~in t[he]~~ diagram. ~~find~~ when $\theta = 35^\circ$, $F = 8$ an[d]

Soln Moment of 'F' about 'P' is $F \times d\,s$...
Here, we have to found the perpendicular ... from P using trigonometry and multi[ply] ... the magnitude of force. When descr[ibing] ... moment you need to give a direction.

$$\text{Moment} = F \times d \sin\theta$$
$$= 8 \times 14 \sin(35^\circ)$$
$$= 64.241 \text{ Nm}.$$ Ans

6N 14N

The diagram shows a set of forces acting
Calculate the Sum of the moments about

Soln Each force is already perpendicular
The moment of the 6N force = $6 \times 2 = 12$ N

The moment of the 14N force = $14 \times 2 = 28$ Nm

The moment of the 5N force = $5 \times (2+3) = 2$

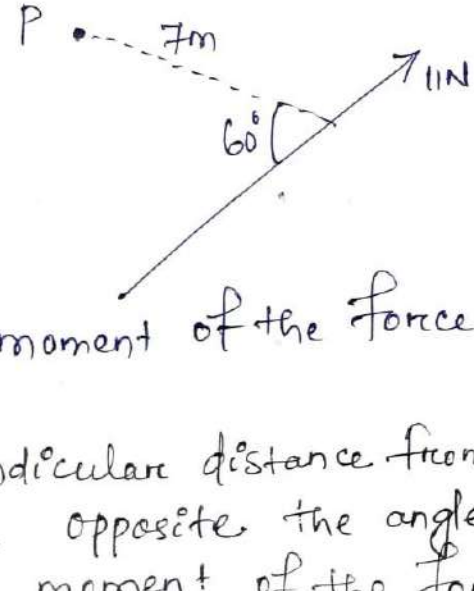
Total clockwise = 28 Nm.

Total Anticlockwise = $25 + 12 = 37$ Nm.

Sum of the moments = $37 - 28 = 9$ Nm. (A

$\therefore$ As the anticlockwise total was greater
anticlockwise as the positive direction

Q finding the moment when the distance
perpendicular.



find the moment of the force about 'P
diagram.

Soln The perpendicular distance from P to the
the distance opposite the angle given b
Therefore the moment of the force, ~~about~~

$11 \times 7 \sin(60^\circ)$

$= 77 \sin 60^\circ = 66.674$

- When the rotation of joint is prevented t
be momentum of resistance will develop.

Displacement prevented → Reactions
Rotation prevented → Moment of r

Types of Support

| 2D Support |

When we provide a Support for plane Structure then it is called 2D Support.

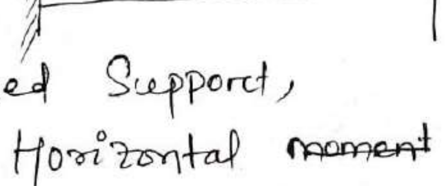
| 3D Support |

When we prov
for Space St
it is called

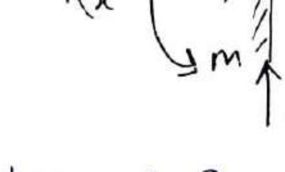
<u>2D Supports:-</u>

① fixed Support
② Hinged Support
③ Roller Support
④ Guided Roller Support → Horizontal (
Sup
→ Vertical g
Suppo

<u>fixed Support:</u>



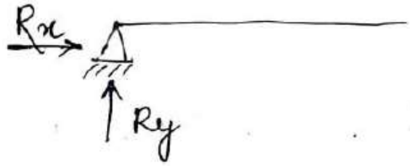
In fixed Support,
Horizontal ~~moment~~ movement. i.e d
prevented



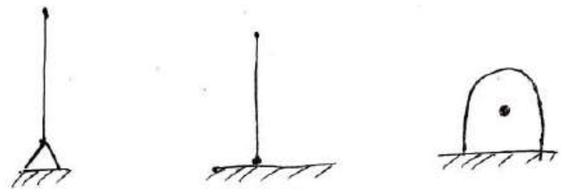
= (3).
(Rx, Ry, M)

Hinged Support:

In Hinged Support rotation (θ) is possi
movement in Horizontal & vertical direc



No. of Reaction = (2)
i.e Rx, Ry.



Reaction in 'x' direction is due to the
of at 'x'.

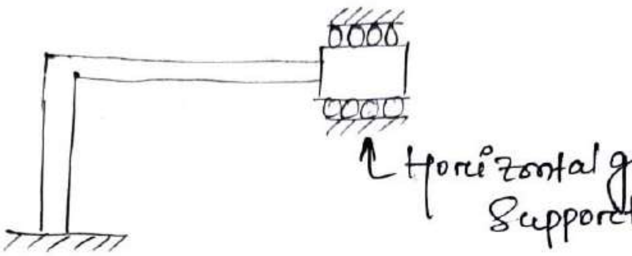
Roller Support:

In Roller Support the movement in horizo
and the rotation are not prevented.
- But the movement in vertical direction is
why there will be a reaction in Vertic
i.e 'Ry' only.

No. of Reaction = (1)

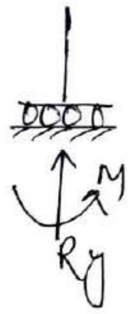
Surface.

Guided Roller Support (fixed Guided Roller S

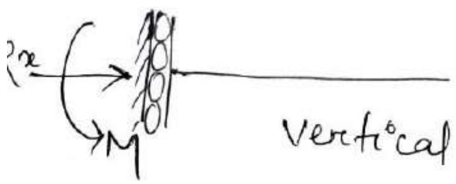


* Here the rotation 'θ' is prevented.
* But the movement in x-direction 'Δx' is
* Moment of 'y' direction 'Δy' is prevented.

So, two reaction will come for guided Rol i.e Ry and Moment of resistance.



No. of Reaction = 2 (Ry, M)
(In case of Horizontal Guided Suppo



No. of reaction = 2 (Rx
Vertical Guided roller Support.

* Horizontal movement is possible in horiz roller Support.
* Vertical Guided Roller Support means is possible in vertical direction.

② Hinge Support

No. of Reaction = ③ (R_x, R_y, R_z)

Because here movement is prevented b
not prevented.

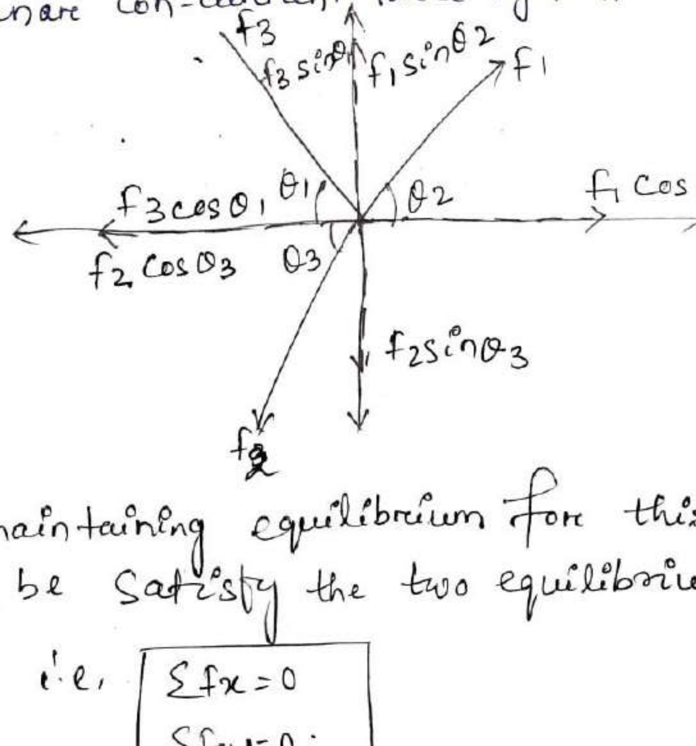
③ Roller Support

No. of reaction = ① & ~~R is~~ Re
to the Surf

- **Conditions of equilibrium:**

Equation of Static equilibrium Conditio
It deals with the Balancin

Types of force system:—
Coplanar Con-current force System:—



For maintaining equilibrium for thi
needs to be satisfy the two equilibriu

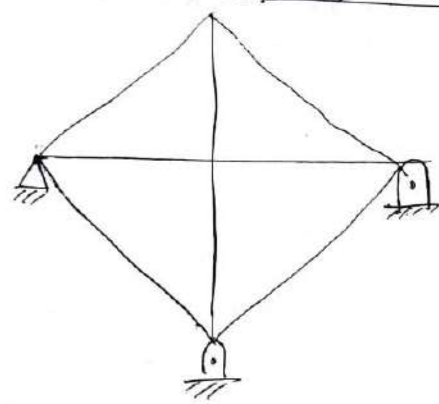
i.e, $\boxed{\begin{aligned}\Sigma f_x = 0 \\ \Sigma f_y = 0\end{aligned}}$

* Here, there is no moment, because the line o
are same and meeting at a point.

$$\boxed{\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma M = 0.}$$

These are three conditions that must be
the equilibrium of Coplanar non-Concurrent

Non-Coplanar Concurrent force System:-



$\Sigma F_x = 0$

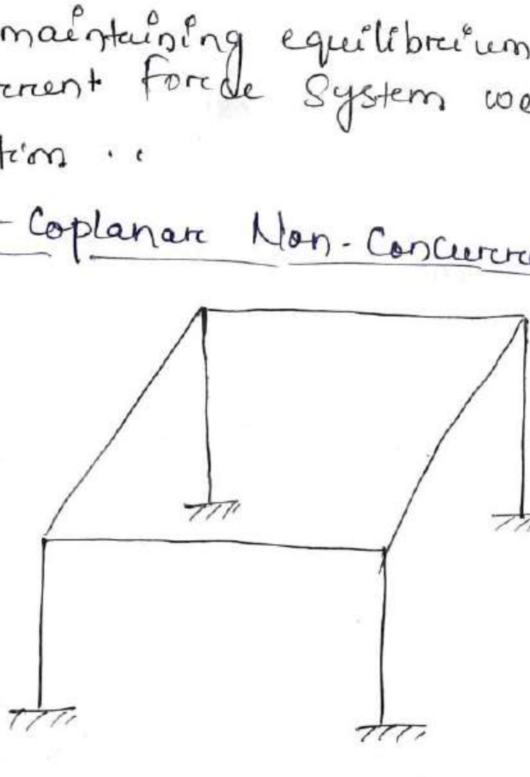
$\Sigma F_y = 0$

$\Sigma F_z = 0$

Pin jointed space truss, Here the internal
only Axial force.

for maintaining equilibrium for non-
Concurrent force System we required
Condition..

Non-Coplanar Non-Concurrent force Syst



This is a
Space fr

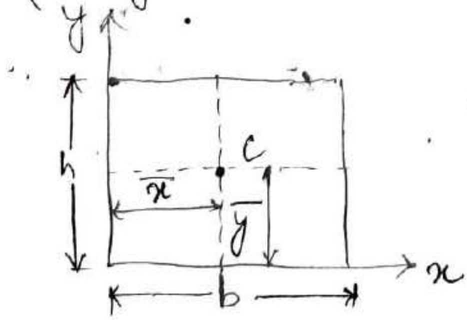
Here, intern
Axial for
Biaxial
Biaxial
Torsion

resultant of a System of parallel force weights of all particles of the body passes
In other words, the point through which of the body acts is known as centre of body has one and only on C.G.

Steps to calculate C.G:-

1. Choose the reference axis (if not given the axis of Symmetry.
2. Divide the given area in to a number defined geometry.
3. Calculate the area and position of C.G from the reference axis.

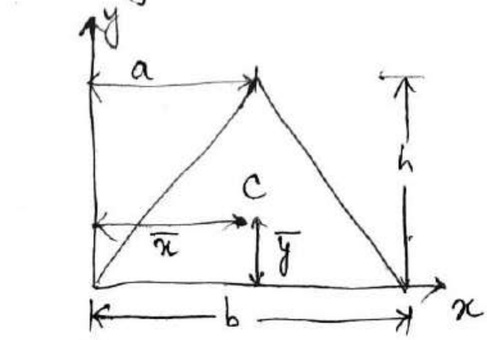
Common shapes	C.G
Rectangle	



$\bar{x} = \frac{b}{2}$

$\bar{y} = \frac{h}{2}$

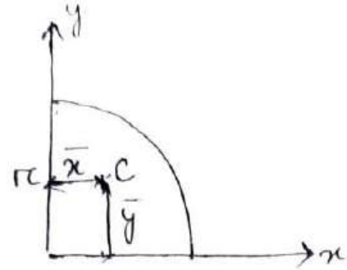
Triangle



$\bar{x} = a$

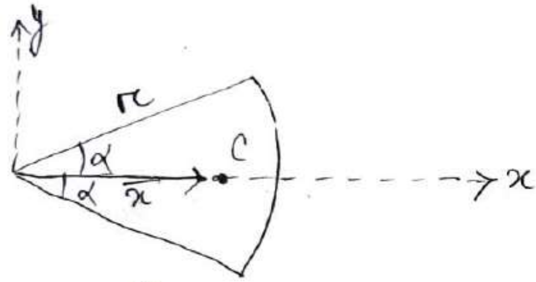
$\bar{y} =$

Quarter circle.



$\bar{x} = \bar{y} =$

Circular Sector.



$\bar{x} =$

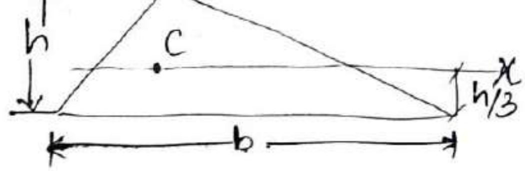
Moment of Inertia (M.I) :—

Moment of inertia is defined as the expressed by the body resisting angul which is the sum of the product of t every particle with its square of a d the axis of Rotation.

i.e. $\boxed{I_{1-1} = \bar{I_x} + A\bar{Y}^2}$

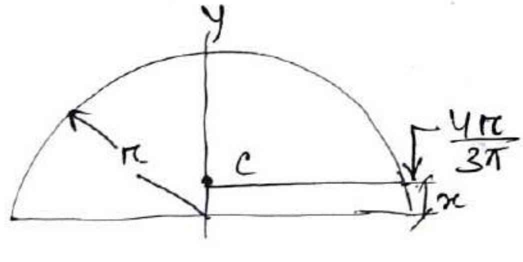
I_{1-1} = moment of inertia of the pl
$\bar{I_x}$ = Sum of MI of the plane fig.
A = Area, $\bar{Y}$ = Difference be axis & the refer

$\boxed{I_{2-2} = \bar{I_{y}} + A\bar{X}^2}$



$A = \frac{1}{2} bh$

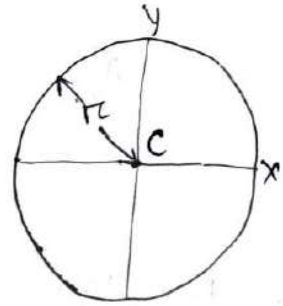
$I_x = \frac{1}{36} bh^3$



$A = \frac{\pi r^2}{2}$

$I_x = \frac{1}{8} \pi r^4$

$I_y = \frac{1}{8} \pi r^4$



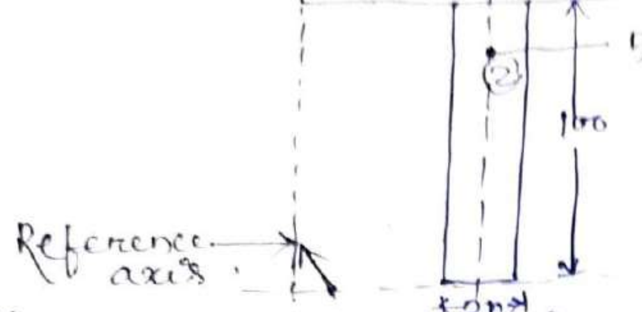
$A = \pi r^2$

$I_x = \frac{1}{4} \pi r^4$

$I_y = \frac{1}{4} \pi r^4$

Free body diagram:

free body diagrams are used to visualiz moments applied to a body and to Calc in mechanics problems. These diagrams ar used both to determine the loading of ind Structural Components and to calculate i forces within a structure.



All dimensions in mm.
Find the C.G. of the T

Sol^n T-section has single axis of symmetry i.e Y
Therefore, Centroid ~~lines Y-Y~~ lies on Y-Y axis.
To find $\bar{y}$, let us divide composite section into t
as shown in figure with areas.

$$A_1 = 100 \times 20 \text{ mm}^2$$

$$A_2 = 20 \times 100 \text{ mm}^2.$$

Considering top most fibre as reference line.
Centroid of A_1 is (0,10) and centroid of A_2 is

$$\bar{x} = 100/2 = 50 \text{ mm}.$$

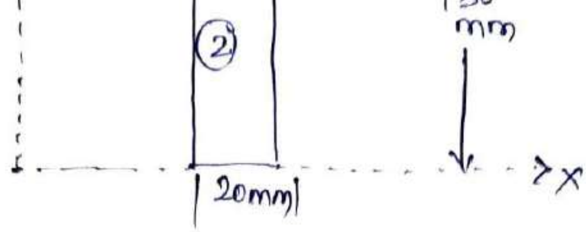
$$y_1 = \cancel{\frac{100+20}{2}} \cancel{= 70\text{mm}}.\ \cancel{100 + \frac{20}{2}}\ \cancel{= 110}\ 10\text{ mm}$$

$$y_2 = \cancel{50\text{mm}}.\ \cancel{60}\ 70 \text{ mm}.$$

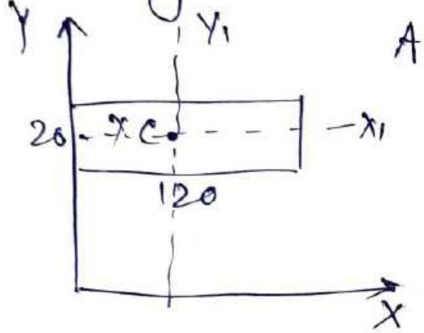
(50,70) centroid

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{100 \times 20 \times 10 + 20 \times 100 \times 70}{100 \times 20 + 20 \times 100}$$

$$= 40 \text{ mm from reference}$$



Rectangle ①



$A = 120 \times 20 = 2400 mm^2$

$x = 60$

$y = 140mm \cdot [(130+10)]$

$I_{x_1} = \frac{bh^3}{12} = \frac{120 \times 20^3}{12} = 80000 mm^4 = 8 \times 10^4 mm^4.$

$I_{Y_1} = \frac{hb^3}{12} = \frac{20 \times 120^3}{12} = 288 \times 10^4 mm^4$

$I_{XX_1} = I_{X_1} + Al^2$

$= 8 \times 10^4 + 2400 \times 140^2$

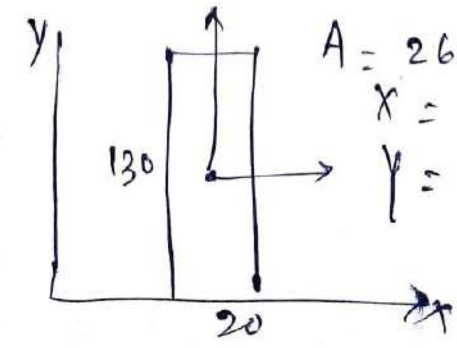
$= 47.12 \times 10^6 mm^4$

$I_{YY_1} = I_{Y_1} + Al^2.$

$= 288 \times 10^4 + 2400 \times 60^2.$

$= 11.52 \times 10^6 mm^4.$

Rectangle ②



$A = 2600 mm^2$

$X = 60mm$

$Y = 65mm.$

$= 3.661 \times 2600 \times 65^2$

$= 14.646 \times 10^6 \ mm^4$

$I_{YY_2} = I_{Y_2} + Al^2$

$= 86.666 \times 10^3 \times 2600 \times 60^2$

$= \cancel{86} \ 9.446 \times 10^6 \ mm^4.$

$I_X = I_{XX_1} + I_{XX_2}$

$= 47.12 \times 10^6 + 14.646 \times 10^6$

$= 61.766 \times 10^6 \ mm^4.$

$I_Y = I_{YY_1} + I_{YY_2}$

$= 11.52 \times 10^6 + 9.446 \times 10^6$

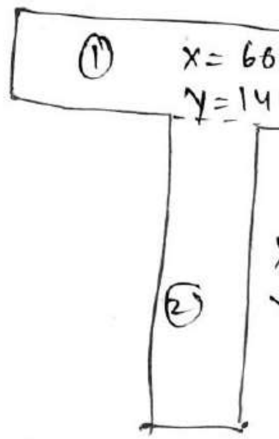
$= 20.966 \times 10^6 \ mm^4.$

<u>Short cut method</u>

I_X = MI of Rectangle 1 + MI of Rectangle 2

$= (I_X + Al^2) + (I_X + Al^2)$

$= \left[\frac{120 \times 20^3}{12} + 2400 \times 140^2\right] + \left[\frac{20 \times 130^3}{12} + 2600 \times 65^2\right]$

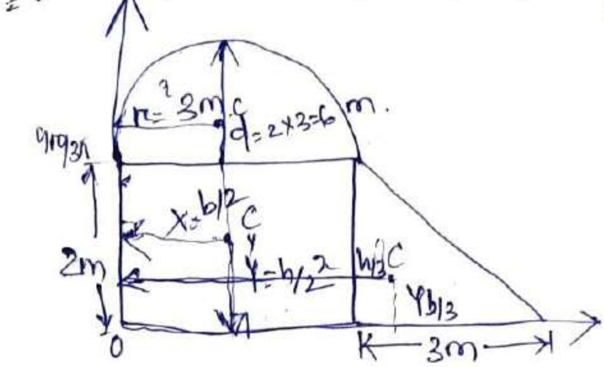


$= 47.12 \times 10^6 + 14.646 \times 10^6$

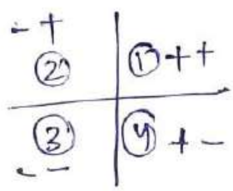
$= 61.766 \times 10^6 \ mm^4.$

$I_y = 20.966 \times 10^6 \, mm^4.$

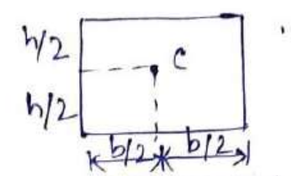
Q Find the centroid of the given shape.



Centroid $c = (\bar{x}, \bar{y})$

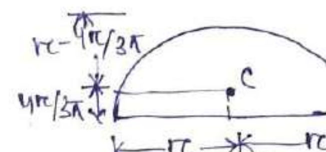


1) Rectangle.

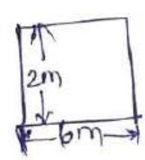
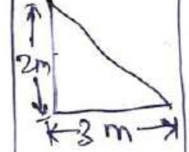
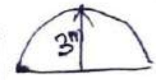


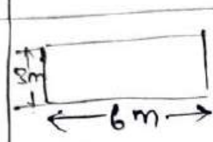
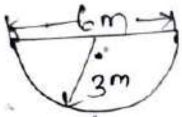
$\boxed{A = b \times h.}$

3) Semicircle.



$\boxed{A = \frac{\pi r^2}{2}}$

S.no.	Shape	Area (m^2)	X (m)	Y (m)	Ax (m^3)	A
1		12	3	1	36	
2		3	7	0.666	21	1
3		~~$\frac{1}{2} \times 6 \times 3$~~ $\frac{\pi r^2}{2} = 14.134$	3	3.273	42.411	
		$\Sigma = 29.137$			$\Sigma = 99.411$	Σ

Slno	Shape	Area (m^2)	X_m	Y_m	$A x_m$ m^3
1		18	3	$1.5+3=4.5$	54
2		14.137	~~1.5+3=4.5~~ 3	$\left(R-\frac{4R}{3\pi}\right)$ 1.726	42.411
		32.137			96.411

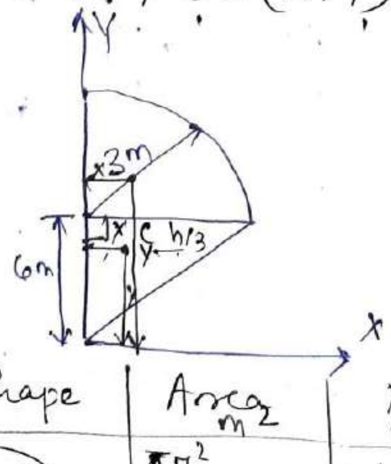
$$\bar{X} = \frac{\Sigma Ax}{\Sigma A} = \frac{96.411}{32.132}$$

$$\boxed{\bar{X} = 3m.}$$

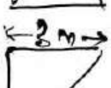
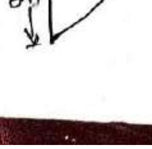
$$\bar{Y} = \frac{\Sigma Ay}{\Sigma A} = \frac{105.4}{32.132} \Rightarrow \boxed{\bar{Y} = 3.279m.}$$

$\therefore$ centroid, $C = (\bar{X}, \bar{Y}) = (3, 3.279)$.

Q



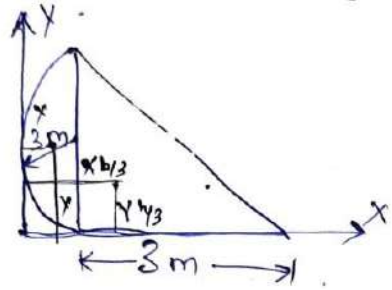
find the centroi

Slno.	Shape	Area m^2	X_m	Y_m	$A x$ m^3
1		$\frac{\pi r^2}{4} = 7.068$	$\frac{4r}{3\pi} = 1.273$	$\frac{4r}{3\pi}+6$ $= 7.273.$	8.99
2		$\frac{1}{2}\times b\times h$ $= 9$	$b/3 = \frac{3}{3} = 1$	$\frac{2h}{3}$ $= \frac{2\times 6}{3}$ $= 4.$	9
		16.068			17.9

$$\boxed{\bar{Y} = 5.439m}$$

$\therefore$ centroid, $C = (\bar{x}, \bar{y})$, $C = (1.12, 5.439$

Q



find centroid.

Slno.	Shape	Area(m^2)	X(m)	Y(m)	A X(m^3)
1	triangle 6, 3	9	$3+\frac{3}{3}=4$	$\frac{6}{3}=2$	36
2	semicircle 3, $x=\left(\pi-\frac{4r}{3\pi}\right)$	14.137	1.726	3	24.4
		23.137			60.4

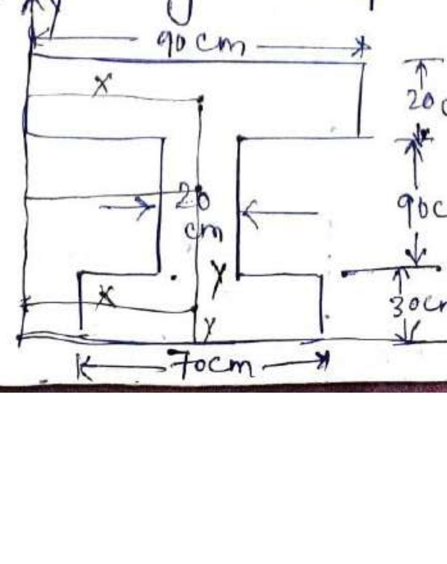
Distance paralle

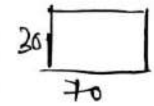
$$\bar{x} = \frac{\Sigma A x}{\Sigma A} = \frac{60.4}{23.137} = 2.61\ m.$$

$$\bar{y} = \frac{\Sigma A y}{\Sigma A} = \frac{60.411}{23.137} = 2.611\ m.$$

$\therefore$ Centroid $(\bar{x}, \bar{y}) = (2.61, 2.611)$.

Q, Locate the Centroid for the given shape w 'x' and Y axis.



3		2100	45	15.	94500
		$\Sigma = 5700$			$\Sigma = 25650$

$$\bar{X} = \frac{\Sigma A x}{\Sigma A} = \frac{256500}{5700} = 45 \text{ cm}$$

$$\bar{Y} = \frac{\Sigma A y}{\Sigma A} = \frac{400500}{5700} = 70.263 \text{ cm}$$

$\therefore$ Centroid, $C = (\bar{x}, \bar{y})$

$$C = (45,\ 70.263)$$

* The centre of gravity or centre of m
Point, where the whole mass of the bod
Concentrated.

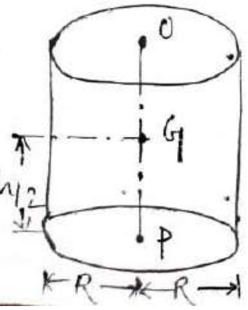
The centroid is the point of the geomet
any object, where the density is who
over the body.

If the body is homogeneous (having co
Its centre of gravity is equivalent to t

Centre of gravity :

1. Cylinder

2. Con



Volume,

$$\boxed{V = \pi R^2 h}$$

$x = R$

$x = R$

$y = R$

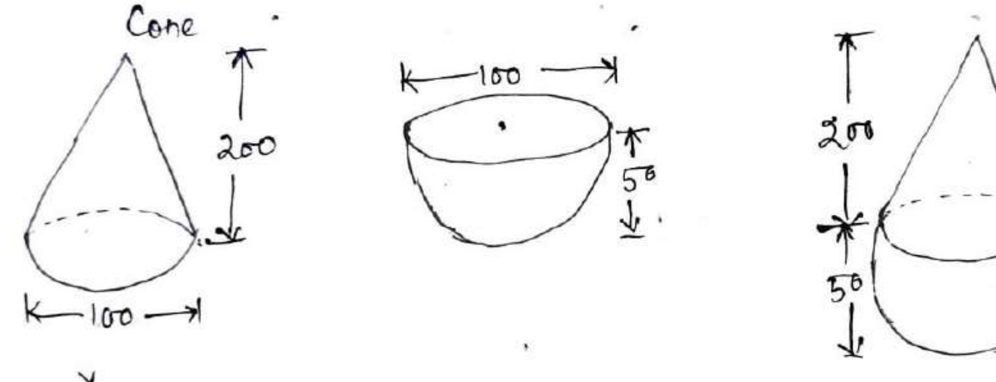
Volume,

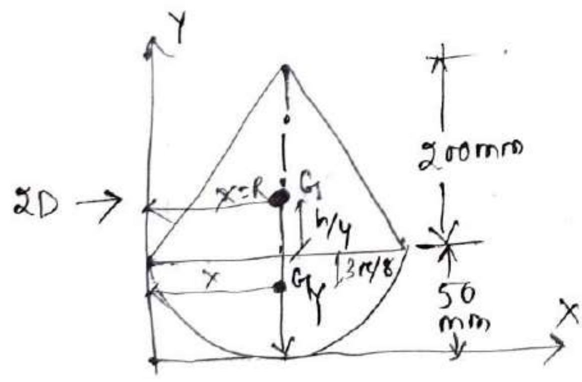
$$\boxed{V = \frac{2}{3}\pi R^3}$$

$x = R$

$y = \frac{3R}{8}$

Q A right circular cone of base diameter 1[...] height 200mm is place on the hemisphe[...] Same diameter. Calculate its centre of [...]



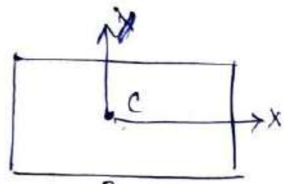


Slno	Volume (mm^3)	X (mm)	Y (mm)	V x (mm^4)	V y (mm^4)
1	$\frac{1}{3} \times \pi \times 50^2 \times 200$ $= 523598.778$ $= 5.235 \times 10^5$	50	100	26179938.75 26.17×10^6	52359877[...] 52.359×10^6
2	$\frac{2}{3} \pi \times 50^3$ $= 261799.387$ $= 2.617 \times 10^5$	50	$50 - 3R/8$ $= 31.25$	13089969.35 13.089×10^6	8181230.844 81.812×10^5
	785398.162			39269908.1	60541108.3

$\therefore$ centre of gravity, $G(\bar{x}, \bar{y})$, (3·

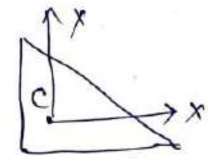
Moment of Inertia:

1. Rectangle



$$\left[\begin{array}{l} I_x = \dfrac{bh^3}{12} \\ I_y = \dfrac{hb^3}{12} \end{array}\right]$$

2. Triangle

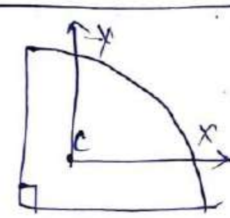


$$\left[\begin{array}{l} I_x = \dfrac{bh^3}{36} \\ I_y = \dfrac{hb^3}{36} \end{array}\right],$$

3.

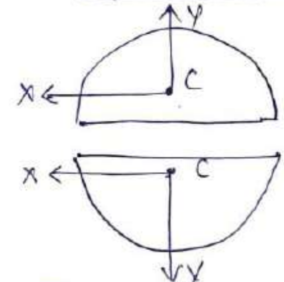
$[I_x =$

4. Quarter circle



$$\left[I_x = I_y = 0.055r^4\right]$$

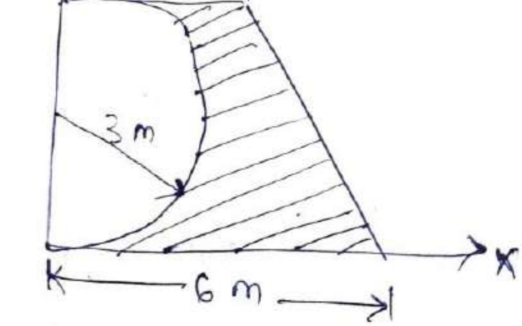
5.1. Semicircle



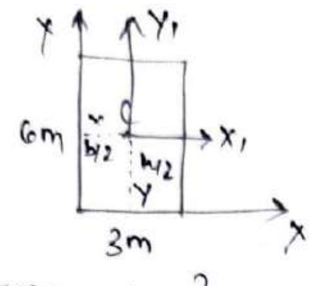
$$\left[\begin{array}{l} I_x = 0.11r^4 \\ I_y = 0.392r^4 \end{array}\right]$$

5.2.

Q. Find the moment of inertia of shaded in figure about x and y axis.



① Rectangle



Area: $18 m^2$

$X = 1.5m$

$Y = 3m$

$I_x = \frac{bh^3}{12} = \frac{3\times6^3}{12} = 54 m^4$

$I_y = \frac{hb^3}{12} = \frac{6\times3^3}{12} = 13.5 m^4$

$I_{xx_1} = I_{x_1} + Al^2 = 54 + 18\times3^2 = \underline{216 m^4}$

$I_{yy_1} = I_{y_1} + AL^2 = 13.5 + 18\times1.5^2 = \underline{54 m^4}$

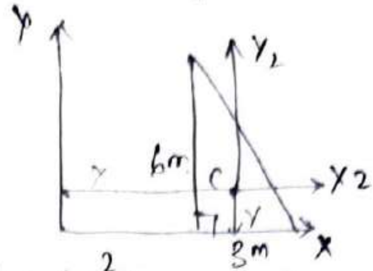
→ parallel axi's theorem

l = Distance between X & X_1 axi's.

" " Y & Y axi's.

$\begin{bmatrix} I_{xx} = L = Y \\ I_{yy} = L = X. \end{bmatrix}$

② Triangle



$A = \frac{1}{2}\times3\times6 = 9m^2$ $X = 4m$. $Y = 2m$.

$I_{x_2} = bh^3/36 = 3\times6^3/36 = 18 m^4$

$I_{y_2} = hb^3/36 = 6\times3^3/36 = 4.5 m^4$

$I_{xx_2} = I_{x_2} + Al^2$

$= 18 + 9\times2^2$

$= \underline{54 m^4}$

$I_{yy_2} = I_{y_2} + Al^2$

$= 4.5 + 9\times4^2$

$= \underline{148.5 m^4}$

③

Shaded part MI ← Unshaded portion should

$I_x = I_{xx_1} + I_{xx_2} - I_{xx_3}$.

$= 216 + 54 - 158.985$

$= \underline{111.015 m^4}$.

$I_y = I_{yy_1} + I_{yy_2} - I_{yy_3}$

$= 54 + 148.5 - 31.819$

$= \underline{170.681 m^4}$.

Ans

$= 216 + 54 - 158.985$

$= \underline{111.015\ m^4}$

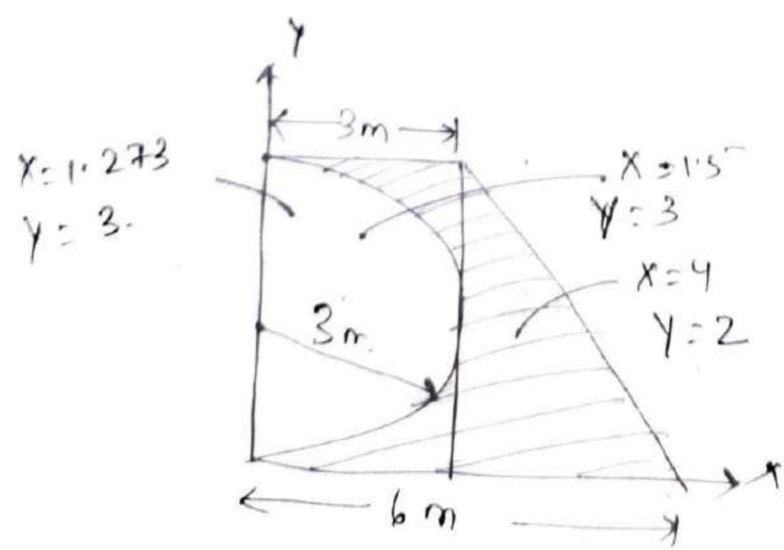
I_Y = MI of ~~Rectangle~~ + MI of Triangle − MI of

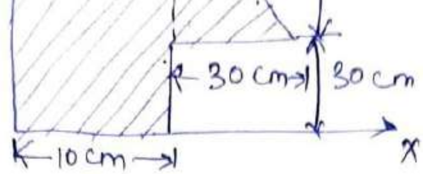
$= (I_Y + Al^2) + (I_Y + Al^2) - (I_y + Al^2)$

$= \left(\frac{6\times3^3}{12} + 18\times1.5^2\right) + \left(\frac{6\times3^3}{36} + 9\times4^2\right) - \left(0.11\times3\right.$

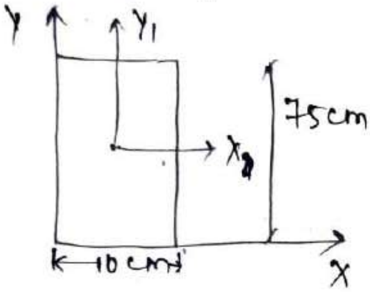
$= 54 + 148.5 - 31.819$

$= \underline{170.681\ m^4}$





Rectangle



$A = 75 \times 10 = 75 cm^2$

$x = 5 cm$

$y = 37.5 cm.$

$I_{x_1} = \frac{bh^3}{12} = \frac{10 \times 75^3}{12} = 351562.5 cm^4$

$I_{y_1} = \frac{hb^3}{12} = \frac{75 \times 10^3}{12} = 6250 cm^4$

$I_{xx_1} = I_{x_1} + Al^2$

$= 351562.5 + 750 \times 37.5^2$

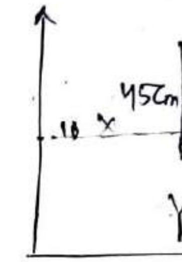
$= 1406250 cm^4$

$I_{xy_1} = I_{y_1} + Al^2$

$= 6250 + 750 \times 5^2$

$= 25000 cm^4$

Triang



$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 30 \times 4$

$x = 20 cm$

$y = 45 cm.$

$I_{x_2} = \frac{bh^3}{36} = \frac{30 \times}{3}$

$I_{y_2} = \frac{hb^3}{36} = \frac{45}{}$

$I_{xx_2} = I_{x_2} +$

$= 75937$

$= 14429$

$I_{yy_2} = I_{y_2} +$

$= 33757$

$= 3037$

$I_x = I_{xx_1} + I_{xx_2} = 1406250 + 1442812.5$

$I_y = I_{yy_1} + I_{yy_2} = 25000 + 303750 = 328$

$= 1406250 + 1442812.5$

$$\boxed{I_x = 2849062.5 \text{ cm}^4}$$

I_y = MI of Rectangle + MI of Triangle

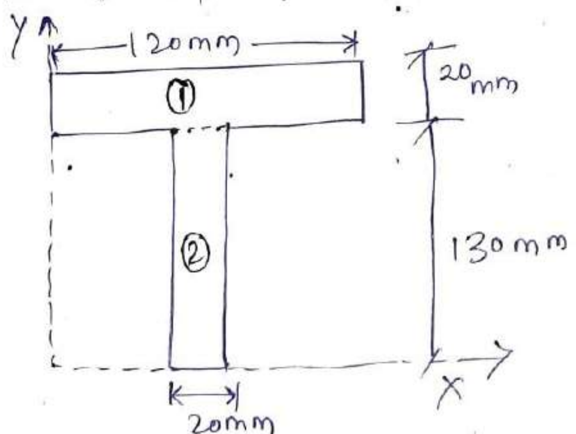
$= (I_y + Al^2) + (I_y + Al^2)$

$= \left[\frac{75 \times 10^3}{12} + 750 \times 5^2\right] + \left[\frac{45 \times 30^3}{36} + 675 \ldots\right.$

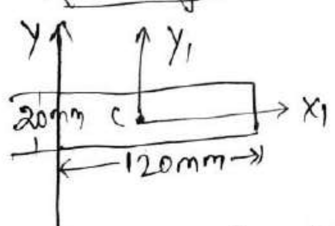
$= 25000 + 303750$

$$\boxed{I_y = 328750}$$

Q. Find the moment of inertia for the gi[ven section] with respect to X and Y axis.



Rectangle - 1



$A = 120 \times 20 = 2400 \text{ mm}^2$

$x = 60$ mm

$y = 130 + 10 = 140$ mm

$I_{x_1} = bh^3/12 = \frac{120 \times 20^3}{12} = 8\ldots$

$I_{y_1} = hb^3/12 = \frac{20 \times 120^3}{12} = 2\ldots$

$I_{xx_1} = I_{x_1} + Al^2 = 8 \times 10^4 + 2400 \times (140)^2 = 47.1\ldots$

$I_{yy_1} = I_{y_1} + Al^2 = 288 \times 10^4 + 2400 \times (60)^2 = \ldots$

Rect[angle - 2]

$$I x_2 = \frac{bh^3}{12} = \frac{20 \times 130^3}{12} = 3.661 \times 10^6 \, mm^4$$

$$I y_2 = \frac{hb^3}{12} = \frac{130 \times 20^3}{12} = 86.66 \times 10^3 \, mm^4$$

$$I xx_2 = I x_2 + Al^2$$
$$= 3.661 \times 10^6 + 2600 \times 65^2$$
$$= \underline{14.646 \times 10^6} \, mm^4$$

$$I yy_2 = I y_2 + Al^2$$
$$= 86.66 \times 10^3 + 2600 \times 60^2$$
$$= \underline{9.446 \times 10^6 \, mm^4}.$$

$$I x = I xx_1 + I xx_2$$
$$= 47.12 \times 10^6 + 14.646 \times 10^6$$
$$= \underline{61.766 \times 10^6} \, mm^4$$

$$I y = I yy_1 + I yy_2$$
$$= 11.52 \times 10^6 + 9.946 \times 10^6$$
$$= \underline{20.966 \times 10^6 \, mm^4}.$$

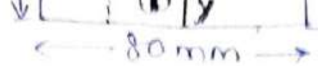
Short cut

$I x$ = MI of Rectangle 1 + MI of Rectangle 2

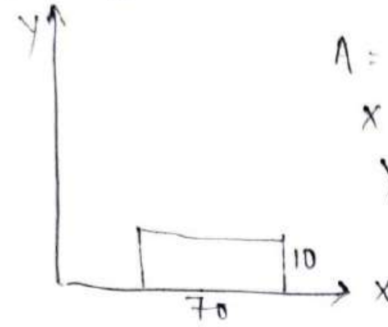
$$= (I x + Al^2) + (I x + Al^2)$$
$$= \left[\frac{120 \times 20^3}{12} + 2400 \times 140^2\right] + \left[\frac{20 \times 130^3}{12}\right.$$
$$= 47.12 \times 10^6 + 14.646 \times 10^6$$
$$\boxed{I x = 61.766 \times 10^6 \, mm^4.}$$

$I y$ = MI of rectangle 1 + MI of rectangle

$$= (I y + Al^2) + (I x + Al^2)$$
$$= \left[\frac{20 \times 120^3}{12} + 2400 \times 60^2\right] + \left[\frac{130 \times 20}{12}\right.$$
$$\boxed{I x = 20.966 \times 10^6 \, mm^4}$$



① Rectangle ①



$A = 700\,mm^2$

$x = 45\,mm.$

$y = 5\,mm.$

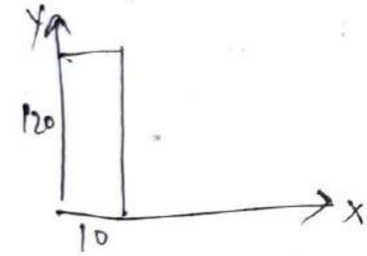
$I_{x1} = \frac{bh^3}{12} = \frac{70 \times 10^3}{12} = 5.8$

$I_{y1} = \frac{hb^3}{12} = \frac{10 \times 70^3}{12} = 2$

$I_{xx_1} = I_{x_1} + Al^2 = 5.833 \times 10^3 + 700 \times 5^2 = 23$

$I_{yy_1} = I_{y_1} + Al^2 = 285.833 \times 10^3 + 700 \times 45^2 = 170$

② Rectangle ②



$A = 1200\,mm^2$ $\quad I_{x2} = \frac{bh^3}{12} = \frac{10}{}$

$x = 5\,mm$

$y = 60\,mm$ $\quad I_{y2} = \frac{hb^3}{12} = \frac{12}{}$

$I_{xx_2} = I_{x_2} + Al^2 = 1.44 \times 10^6 + 1200 \times 60^2 = 5.7$

$I_{yy_2} = I_{y_2} + Al^2 = 10 \times 10^3 + 1200 \times 5^2 = 40 \times$

$I_x = I_{xx_1} + I_{xx_2} = 23.333 \times 10^3 + 5.76 \times 10^6 =$

$I_y = I_{yy_1} + I_{yy_2} = 1703.333 \times 10^3 + 40 \times 10^3 =$

Short cut $\quad I_x =$ MI of rectangle ① + MI of

$= I_{xx_1} + I_{xx_2}$

$= (I_{x_1} + Al^2) + (I_{x_2} + Al^2)$

$= \left[\frac{70 \times 10^3}{12} + 700 \times 5^2\right] + \left[\frac{10 \times 120^3}{12} + 1200\right.$

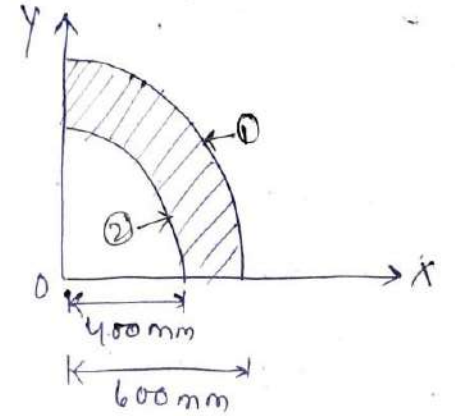
$= (I_{y_1} + Al^2) + (I_{y_2} + Al^2)$

$= \left[\frac{10 \times 70^3}{12} + 700 \times 45^2\right] + \left[\frac{120 \times 10^3}{12} + 120\right.$

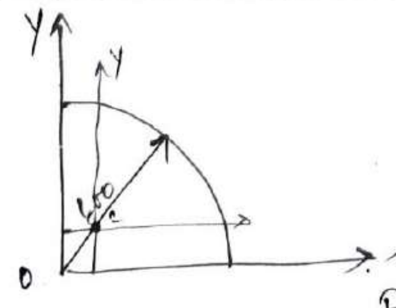
$= 1703.333 \times 10^3 + 40 \times 10^3$

$\boxed{I_y = 1.7433 \times 10^6 \ mm^4}$.

Q. find the moment of inertia with respe
X and Y axis for the shaded region



(i) MI of Quarter circle (1)



$A = \frac{\pi r^2}{4} = 282.74 \times 10^3$

$X = Y = \frac{4r}{3\pi} = \frac{4 \times}{3}$

$I_{X_1} = 0.055 r^4 = 0.05$

I_{XY}

$I_{XX_1} = I_{X_1} + Al^2 = 7.128 \times 10^9$

$\boxed{I_{XX_1} = 2.546 \cdot 10^{10} \ mm^4.}$

$I_{x_2} = 0.035 r^4 = 0.055 \times 400^4 =$

$= I_{y_2}$

$I_{xx_2} = I_{x_2} + A l^2$

$= 1.408 \times 10^9 + 125.663 \times 10^3 \times 16$

$= 5.029 \times 10^9 \ mm^4 = I_{yy_2}$

$\therefore$

$I_x = I_{xx_1} - I_{xx_2} = 2.546 \times 10^{10} - 5.029 \times 10^9$

$= \underline{2.0431 \times 10^{10} \ mm^4} = \underline{I_y}$

$I_y = I_{yy_1} - I_{yy_2} = \underline{2.0431 \times 10^{10} \ mm^4}$

<u>Short cut</u>

$I_x =$ MI of Quarter circle (1) − MI of Quart

$= (I_{x_1} + A l^2) - (I_{x_2} + A l^2)$

$= \left[0.055 \times 600^4 + \frac{\pi \times 600^2}{4} \times \cancel{254.649} \left(\frac{4 \times 600}{3\pi}\right)^2\right] -$

$\left[0.055 \times 400^4 + \frac{\pi \times 400^2}{4} \times \left(\frac{4 \times 400}{3\pi}\right)^2\right]$

$= 2.546 \times 10^{10} - 5.029 \times 10^9$

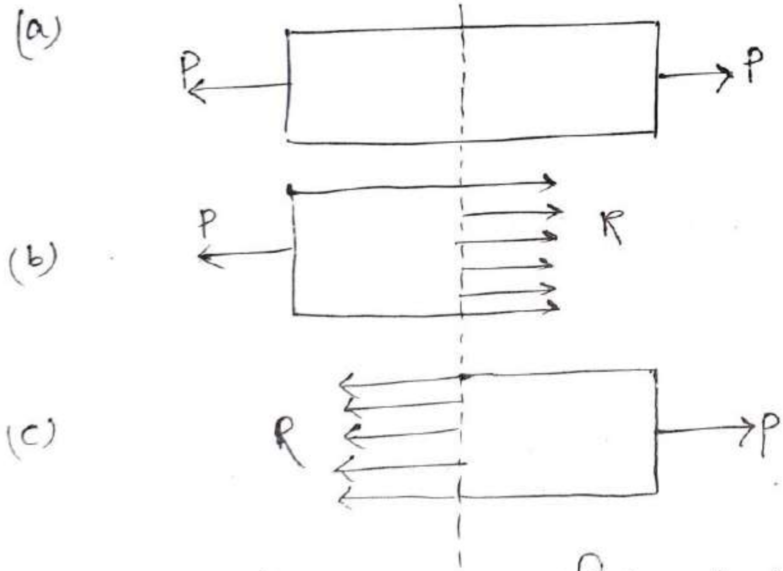
$$\boxed{I_x = 2.043 \times 10^{10} \ mm^4 = I_y}$$

→ The external force acting on the body ...

→ In a single line, load is applied on ... Stress is ~~included~~ induced in the ma... body.

→ Stress is denoted by 'σ' (sigma).

→ Let a rod of uniform cross-sectiona... subjected to pulling force (P), to res... then an internal force (R), will be indu... material.



∴ St...

$\boxed{\sigma = }$

Unit =

$\frac{N}{m^2} =$

R = P → (no tearing of the body)

(Applied force = resisting force)

$$\left[\text{Stress} = \frac{\text{internal resisting force}}{\text{Area}} \right]$$

Units of Stress:

1 pascal = $1\,N/m^2$

1 kilo pascal = $10^3\,N/m^2$

1 mega pascal = $10^6\,N/m^2$

1 giga pascal = $10^9\,N/m^2$.

Kilo = 10^3

mega = 10^6

Giga = 10^9

Tera = 10^{12}

* $1\,KN = 10^3\,N$.
* $1\,N = 10^{-3}\,KN$.

we know, $1\,mpa = 10^6\,N/m^2$

$$= 10^6 \times \frac{10^{-3}\,KN}{10^6\,mm^2}$$

$$= 10^{-3}\,KN/mm^2$$

$$= 10^{-3} \times 10^3\,N/mm^2.$$

$$\boxed{1\,mpa = 1\,N/mm^2.}$$

Strain:

This is the ratio of change in di
the Original dimension.

→ It is denoted by 'e'.

$\Rightarrow \boxed{e = \frac{\Delta l}{l}} \Rightarrow \frac{\text{Change in length}}{\text{Original Length}} \Rightarrow \frac{\Delta d}{d}$

l = linear Strain → Tensile
→ Compressive.

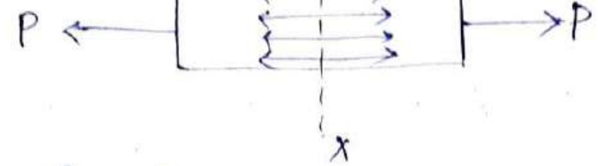
d = Lateral Strain

v = Volumetric Strain.

Types of Stress:

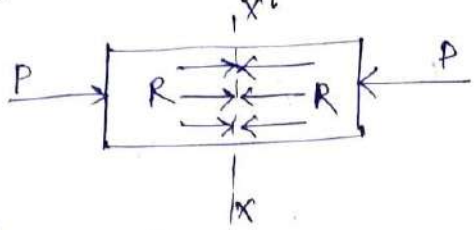
A material is capable of offering th
Stresses;

(i) Tensile Stress
(ii) Compressive stress.
(iii) Shear Stress.



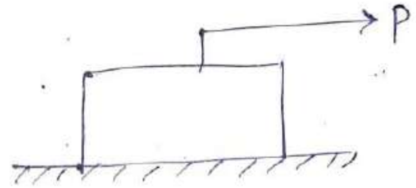
(ii) Compressive Stress :-

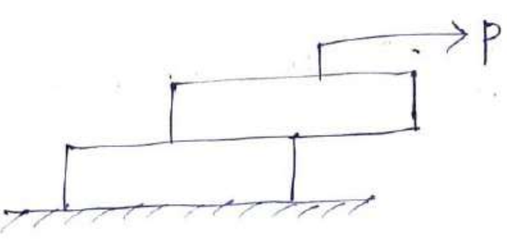
when the offering resistance by a member is against a decrease in length ... Said to offer a Compressive Stress.



(iii) Sheare Stress :- (τ)

A Load P is applied tangentially of the face of the body of which, bottom ... Such force acting tangentially along a ... called as shear ~~force~~ Stress.





→ The resistance provided in this case is ... resistance.

→ Shear Stress is defined as the ratio ... resistance to the shear area.

Mathematically, $\boxed{\tau = \frac{R}{A}}$

$\boxed{e = \frac{\Delta l}{l}}$ → here, Δl → change in l

(ii) Compressive Strain:

The ratio of decrease in length to length, is called Compressive Strain

$\boxed{e = \frac{\Delta l}{l}}$ → here, Δl → change in l

(iii) Lateral Strain:

The ratio of the change in lateral d Original lateral dimension, is called

$\boxed{e = \frac{\Delta d}{d}}$ → Here, Δd → cha di

(iv) Volumetric Strain:

The ratio of the change in Volu Original Volume, is called as Volume

$\boxed{e = \frac{\Delta v}{v}}$ → Here Δv = cha

Imp

Hook's Law: –

It States that "when a materi Such that the intensity of Stress with limit, the ratio of the intensity of the Corresponding Strain is a Con

$$\frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{e} = \text{Constant}$$

Modulus of elasticity (or) Young's Modu...

In case of axial loading, the ratio of tensile or compressive stress to the strain is constant.

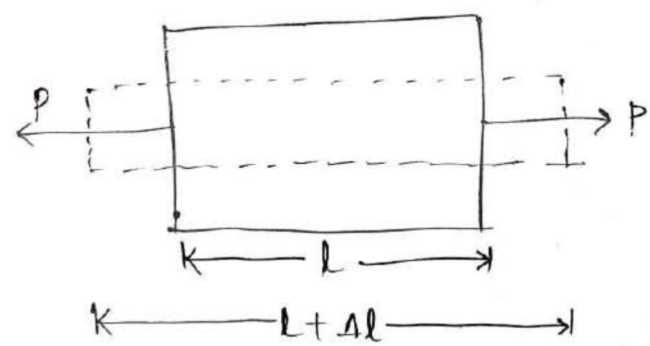
→ this ratio is called modulus of elast... modulus and is denoted by 'E'.

$$\frac{\sigma}{e} = \text{Constant} = E.$$

$\Rightarrow \boxed{E = \frac{\sigma}{e}}$ [∵ Constant is noth... young's modul...

Deformation of prismatic bar due to ...

Throughout the length no change in dimension or no change in cross-sectional area.



Consider a prismatic bar is su... tensile load (P) → Same as an ...

The stress will be introduced due ... given by, $\boxed{\sigma = \frac{P}{A}}$.

We know, Strain; $e = \frac{\Delta l}{l}$

According to Hook's law. 6...

→ In the above formula, the term A
axial rigidity. [AE → Axial rigi

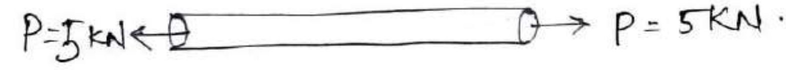
Questions

(1) A load of 5KN is to be raised with
wire. find the minimum diameter of a
stress is not to exceed 100 Mpa.

Given: Stress (σ) = 100 Mpa

$= 100 \times 10^6 \ N/m^2$

External force (P) = 5 KN = 5×10^3



$$\sigma = \frac{P}{A}$$

$$\Rightarrow 100 \times 10^6 = \frac{5 \times 10^3}{\frac{\pi}{4} d^2}$$

$$\Rightarrow d^2 = \frac{5 \times 10^3}{100 \times 10^6 \times \frac{\pi}{4}}$$

$$\Rightarrow d = \sqrt{\frac{5 \times 10^3}{100 \times 10^6 \times \frac{\pi}{4}}}$$

$$\Rightarrow \boxed{d = 7.97 \times 10^{-3} m}$$

$$\Rightarrow d = 7.97 \times 10^{-3} \times 10^3 mm$$

$$\Rightarrow \boxed{d = 7.97 \ mm.}$$

Given; $l = 500\,mm$

$P = 300\,KN = 300 \times 10^3\,N$

$E = 2 \times 10^5\,N/mm^2$

$A = 20mm \times 10mm = 200\,mm^2.$

$$\therefore \Delta l = \frac{Pl}{AE} = \frac{300 \times 10^3 \times 500}{200 \times 2 \times 10^5} = 3.75\,mm$$

$$\text{Strain} = \frac{\Delta l}{l} = \frac{3.75}{500} = 7.5 \times 10^{-3}.$$

3. A hollow cylinder 2m long has an outside 50mm and inside diametere of 30mm. is carrying a load of 25 KN, find the St cylinder, also find deformation of the c

Take $E = 100\,GPa$.

Given; $L = 2m = 2 \times 1000\,mm = 2000\,mm.$

$D = 50\,mm$

$d = 30\,mm.$

$E = 100\,GPa$

$P = 25\,KN = 25 \times 10^3\,N.$

$\therefore$ Area of hollow cylinder $= A = \frac{\pi}{4}(D^2$

$$\Rightarrow A = \frac{\pi}{4}\left((50)^2 - (30)^2\right)$$

$$\Rightarrow \boxed{A = 1256\,mm^2}$$

$E = 100\,GPa$

$= 100 \times 10^9\,N/m^2.$

$= 100 \times 10^9\,N/10^6$

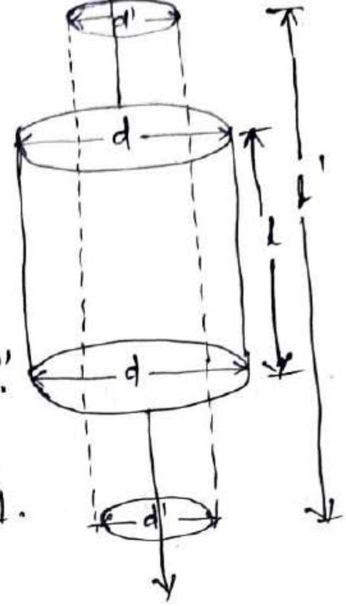
$= 100 \times 10^9 \times 10^6\,N/mm^2.$

$E = 100 \times 10^3\,N/mm^2$

$1m \rightarrow$

$1m^2 \rightarrow$

$\boxed{1m^2 =}$



linear strain $= + \frac{l'}{l}$

lateral strain $= - \frac{d'}{d}$

→ poisson's ratio is defined as "the ratio between the lateral strain to the longitudinal strain."

→ It is denoted as μ or $\frac{1}{m}$ and the value of μ is always less than 1.

Mathematically,

$$\mu = \frac{-\text{lateral strain}}{\text{longitudinal strain}}$$

→ It is a constant and it has no unit.

→ ex: (i) poisson ratio of concrete = 0.15 to 0.3.

(ii) poisson ratio of steel = 0.27 to 0.3.

→ poisson ratio varies from $\boxed{-1 \text{ to } 0.5}$

Elastic Constant

E: Young's modulus of elasticity

K: Bulk Modulus

C: Shear modulus

(i) Young's modulus (E):

It is defined as "the ratio of tensile stress to the corresponding strain."

mathematically, $\boxed{E = \frac{\sigma}{e}}$.

Mathematically, $\boxed{K = \frac{\sigma}{e_v}}$ $\left[\because e_v = \frac{\Delta v}{V}\right.$ volum

(iii) Shear Modulus (C or G):

It is defined as "the ratio of shear st
Shear strain."

→ Higher the value of shear modulus, shows
body is highly rigid.

mathematically, $\boxed{C = \frac{\tau}{\varepsilon}}$

where, τ = shear stress
ε = Shear Strain.

* Relationship between Elastic Constants E,

(i) Relationship between E and K:

$$\boxed{E = 3K\left(1 - \frac{2}{M}\right)}$$

(ii) Relationship between E and C:

$\boxed{E = 2C\left(1 + \frac{1}{M}\right)}$ where, $\frac{1}{M}$ is p

(iii) Relationship between E, K and C.

$$\boxed{E = \frac{9KC}{3K + C}}$$

* Proof: $\boxed{E = \frac{9KC}{3K + C}}$

We know; $E = 3K\left(1 - \frac{2}{M}\right) \Rightarrow \boxed{1 - \frac{2}{M} = \frac{E}{3K}}$

Now adding (1) and (3)

$$1 - \cancel{\frac{2}{M}} + 2 + \cancel{\frac{2}{M}} = \frac{E}{3K} + \frac{E}{C}$$

$$\Rightarrow 1 + 2 = \frac{E}{3K} + \frac{E}{C}$$

$$\Rightarrow 3 = \frac{EC + E3K}{3KC}$$

$$\Rightarrow 3(3KC) = EC + E3K.$$

$$\Rightarrow 9KC = E(C + 3K)$$

$$\Rightarrow \boxed{E = \frac{9KC}{3K+C}}$$ Hence proved.

Questions:

(1) The modulus of rigidity of a material ... find the poisson's ratio if the modulus ... of that material is $2.1 \times 10^5 N/mm^2$.

Given; $C = 0.8 \times 10^5 N/mm^2$

$E = 2.1 \times 10^5 N/mm^2$.

μ or $\frac{1}{M} = ?$

$$\therefore E = 2C\left(1 + \frac{1}{M}\right)$$

$$\Rightarrow 2.1 \times 10^5 N/mm^2 = 2 \times 0.8 \times 10^5 N/mm^2 \ ...$$

$$\Rightarrow \frac{2.1 \times 10^5}{2 \times 0.8 \times 10^5} = 1 + \frac{1}{M}$$

$$\Rightarrow \frac{2.1 \times 10^5}{2 \times 0.8 \times 10^5} - 1 = \frac{1}{M}$$

to an axial pull of 3600N. It was found lateral dimension of the rod changed to 5.9991 mm. find the poisson's ratio an elasticity of that rod.

Given; $C = 0.8 \times 10^5 \ N/mm^2$

$A = 6mm \times 6mm$

$= 36 mm^2$

$P = 3600 N$

$$\sigma = \frac{P}{A} = \frac{3600}{36} = 100 \ N/mm^2$$

Longitudinal strain, $E = \frac{\sigma}{e} \Rightarrow \boxed{e = \frac{\sigma}{E}}$

Lateral strain = $\frac{\text{Change in lateral dimens}}{\text{Original dimension}}$

$$= \frac{6 - 5.9991}{6} = 0.00015.$$

Poisson's ratio:

$$\frac{1}{\mu} = \frac{\text{lateral strain}}{\text{Longitudinal strain}}$$

$$\Rightarrow \frac{1}{\mu} = \frac{0.00015}{\frac{\sigma}{E}}$$

$$\Rightarrow \frac{1}{\mu} = \frac{0.00015}{100} \times E$$

$$\Rightarrow \frac{1}{E\mu} = \frac{0.00015}{100}$$

$$\Rightarrow \mu E = \frac{100}{0.00015} \Rightarrow \boxed{\mu E = \frac{2 \times 10^6}{3}}$$

Equating equation (1) and (2)

$$\Rightarrow 2\times 0.8\times 10^5(m+1) = \frac{2\times 10^6}{3}$$

$$\Rightarrow m+1 = \frac{2\times 10^6}{3(2\times 0.8\times 10^5)}$$

$$\Rightarrow m = \frac{2\times 10^6}{3(2\times 0.8\times 10^5)} - 1$$

$$\Rightarrow \boxed{m = 3.167}$$

$$\because \mu = \frac{1}{m} = \frac{1}{3.167}$$

$$\Rightarrow \boxed{\mu = 0.315}$$

$$\therefore E = 2C \quad 1+\frac{1}{m}$$

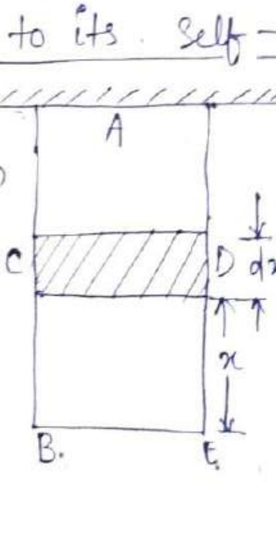
$$\Rightarrow E = 2\times 0.8\times 10^5 \left(1 + 0.315\right)$$

$$\Rightarrow E = 210400$$

$$\Rightarrow \boxed{E = 2.1\times 10^5 \ N/mm^2}$$

Axial deformation of bar due to its self-

Considering a prismatic bar of uniform cross-section of length 'l' suspended freely.



weight of the bar $= W$

Unit weight of the bar $= \gamma$

$$\gamma = \frac{W}{\text{Volume}}$$

$$\Rightarrow \gamma = \frac{W}{Ax}$$

A = Area
x = length
$Ax \rightarrow$ Volume.

Weight of bar below element dx

$$\Rightarrow W_x = \gamma(Ax)$$

Elongation of elemental length dx

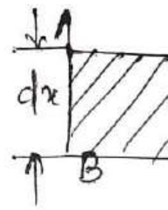
$$\Rightarrow \boxed{d\Delta = \frac{W_x\,dx}{AE}}$$

Let E, ρ, γ, be the Young's modulus, dens
unit weight of the body respectively.

We know, (Volume) $V_x = A \times x$

$$\gamma = \frac{W_x}{V_x}$$

$$\Rightarrow \boxed{W_x = \gamma \times V_x}$$



Now, consider a strip ABCD of length by the deformation for the strip ABCD $= \frac{W_x\,dx}{AE}$.

Total deformation of the bar $= \int_0^l \frac{w_x \times dx}{AE}$

$$= \int_0^l \frac{\gamma \times V_x}{AE}\,dx$$

$$= \int_0^l \frac{\gamma \times A \times x}{AE}\,dx$$

$$= \int_0^l \frac{\gamma x}{E}\,dx$$

$$\Rightarrow \frac{\gamma}{E}\int_0^l x\,dx$$

[∵ γ and E a

$\Rightarrow \boxed{\Delta = \frac{\gamma L^2}{2E}}$ Δl: Total deformat

Notes

(1) Ultimate Stress = $\frac{\text{Maximum Load}}{\text{Area}}$

(2) Yield Stress = $\frac{\text{yield point Load}}{\text{Area}}$

(3) Safe Stress = $\frac{\text{yield Stress}}{\text{Load factor}}$

Mechanical properties of materials:-

(1) Rigidity:

- This is defined as the property posse body to change its shape.
- It means when an external force is material, there won't be any cha due to itermolecular attraction by th particles.
- this is the property of the material t bending.

(2) Elasticity:—

- This is the property of a body by vi return its original shape after re external force Causing deformation on it.

- The ability of a material to retain the chang[e] application of load is known as plasticity
- Plastic deformation is the property of du[ctile] malleable Solids.

(4) Compressibility :-

- this is the property of material by virtu[e] it tends to flatten and reduce in size
- This nature or property of material chan[ges] molecular Structure of the material.

(5) Hardness :-

- the property of material by virtue of [...] the local Surface deformation when unde[r] drilling, impacts etc.
- It is the State of material, being hard [...] can withstand friction.

(6) Toughness :-

- the amount of energy per unit volume th[at] Can absorb before rupture is called [...]
- It can be defined as the ability of a ma[terial to] resist breaking when force is applie[d]
- this property allows the material to de[form ...] rupture or fracture.

(7) stiffness :-

- The property of material which resists c[...] when a force is applied to it. This is th[e ...] a material.
- The material having more flexibility ha[s ...]
- A stiff material has high long modulus

Ex- Bone, Concrete, ceramic, cast ir

(9) Ductility:

- It is defined as the ability of a mater permanent deformation through elongati in cross-sectional area or bending at ro without fracturing.
- This is an ability to undergo last deformation in tension.

Ex- Copper, aluminium, steel.

(10) Malleability:

- This is the property of material by hammered ~~to~~ into a thin sheet wi

Ex- lead, tin, gold, silver, aluminiu

(11) Creep:

- This is the permanent change in shape material which increases as a functio application of load and elevated tem
- Creep is time independent.
- Creep begins at different tempera different material.

(12) Fatigue:

This is the deteriation of the mater repeated cycle of stress and strain re progressive cracking, eventually pro

(13) Durability:

(i) The ability of a material to remain Se during the useful time without damagi material.

(ii) It represents how long the material work

(14) Tenacity:

The property of material to resist the brea as tenacity.

Notes

(1) Percentage of elongation:-

→ It is a measure of ductility.

→ This can be obtained as =

$$\boxed{\frac{\text{final Length} - \text{Initial Length}}{\text{Initial Length.}}}$$

Or, $\boxed{\frac{\Delta l}{l} \times 100.}$

where, Δl = Change in length.

(2) Percentage of reduction in area:-

→ This is the measure of the Specimen tha that Specimen narrowed when it underg Load application.

→ It is obtained as follows:

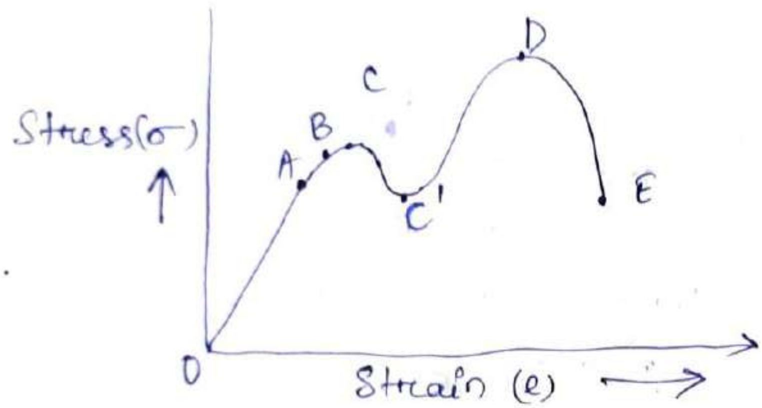
$$\boxed{\frac{\text{final area} - \text{Initial area}}{\text{initial area}} \times 100}$$

the material is more ductile.

(iii) Percentage elongation is a measure of

(iv) percentage reduction area is also a m
ductility.

- <u>Stress-Strain diagram for mild steel:</u>



OA → proportional limit

AB → Elastic limit

CC' → Yield point (Upper yield point
yield point C')

C'D → Ultimate stress point (point D)

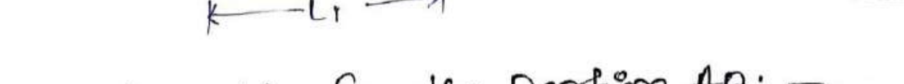
DE → Breaking point (Point E).

→ This curve is obtained when a mild steel
undergoes a tensile test. The plot from
is a straight line, this portion obeys
and the straight line is called the <u>lin</u>
<u>proportionality</u>.

→ In this range of extension, the stress is
to strain i.e $\boxed{\sigma \propto e}$.

→ Again if the Specimen is extended beyo
limit, plastic deformation Occurs.
→ In the range of 'B' to 'C' strain incre
increase in stress.
→ At point 'C' the material goes extended
in load and the Stress at point 'C' is
yield point.
→ At point 'C'' the material again offers r
greater extension and the Stress Corre
point is called Lower-yield point."
→ As the load is increased, the extension
the point 'D' indicates the necking o
and the stress Corresponding to this po
ultimate tensile Stress.
→ As the extension is increased, the load reg
and the Specimen breaks at the point of
point is called Stress failure.

Elongation in bar of varying Section:-
(changes in cross-secti



The elongation for the portion AB:-

$$\boxed{\Delta l_1 = \frac{PL_1}{A_1 E}}$$

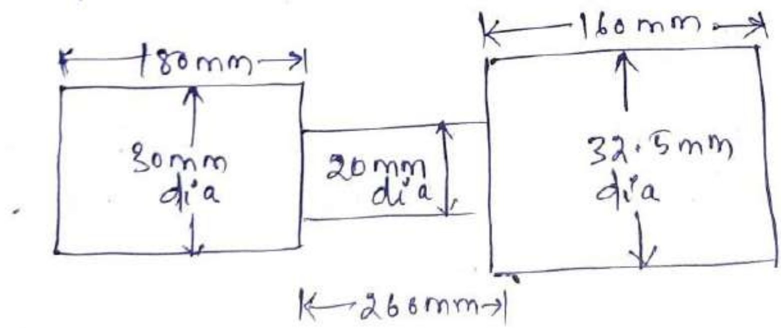
$$\Delta l_3 = \frac{Pl_3}{A_3 E}$$

So total elongation; $\boxed{\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3}$

$$= \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E}$$

$$= \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)$$

Q A bar consisting of 3 lengths. Find th three parts and the total elongation of axial pull of 40KN.
Take Young's modulus 2×10^5 N/mm^2.



Ans Given; $P = 40\,kN = 40 \times 10^3\,N$

$E = 2 \times 10^5$ N/mm^2

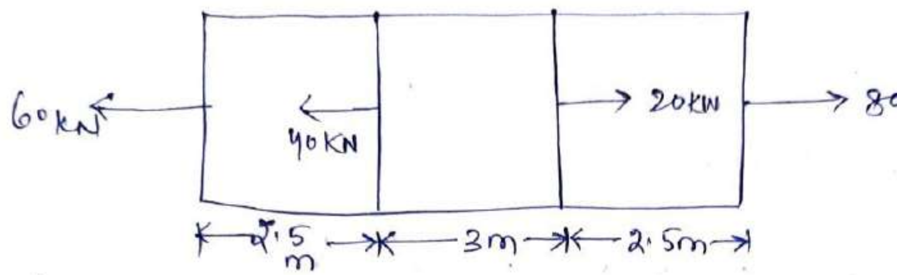
$A_1 = \frac{\pi}{4}(30)^2 = 706.85\ mm^2$

$A_2 = \frac{\pi}{4}(20)^2 = 314.15\ mm^2$

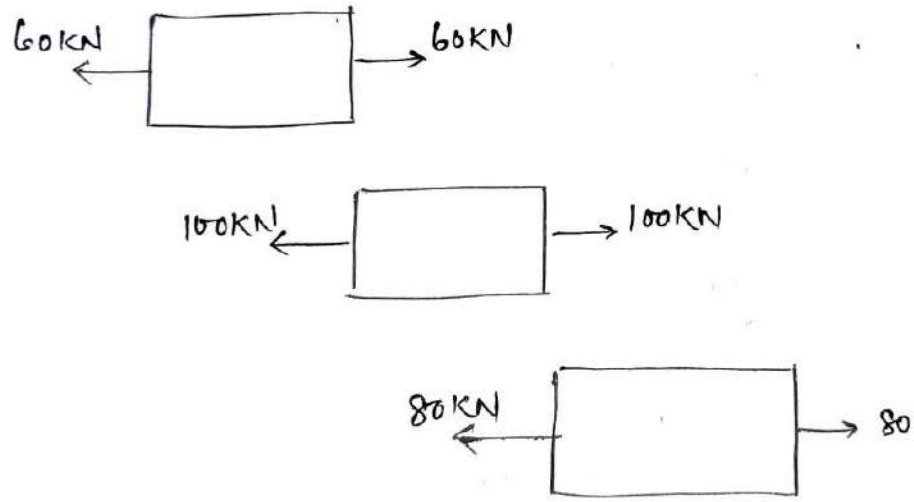
$A_3 = \frac{\pi}{4}(32.5)^2 = 829.579\ mm^2$

$$\Delta l_1 = \frac{40 \times 10^3 \times 180}{706.85 \times 2 \times 10^5} = 0.50\,mm$$

8m long is Subjected to forces as shown below. find the total deformation if of the bar is 200 GPa. All force are in K



free body diagram



$P_1 = 60KN = 60 \times 10^3 N$ $\quad L_1 = 2.5 m = 2.5$

$P_2 = 100KN = 100 \times 10^3 N$ $\quad L_2 = 3m = 3 \times 10$

$P_3 = 80KN = 80 \times 10^3 N$ $\quad L_3 = 2.5 = 2.5$

$E = 200 GPa$

$= 200 \times 10^9 N/m^2$ $\quad A = 1000 mm^2$

$= 200 \times 10^9 N/10^6 mm^2$

$= 200 \times 10^{9-6} N/mm^2$

$= 200 \times 10^3 N/mm^2$.

$$\Delta l_3 = \frac{P_3 l_3}{AE} = \frac{80 \times 10^3 \times 2.5 \times 10^3}{1000 \times 200 \times 10^3} = 1\,mm.$$

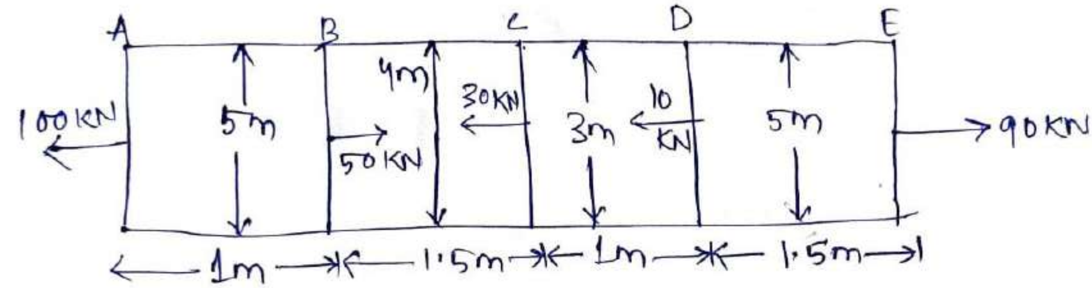
$\therefore$ Total elongation

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$$

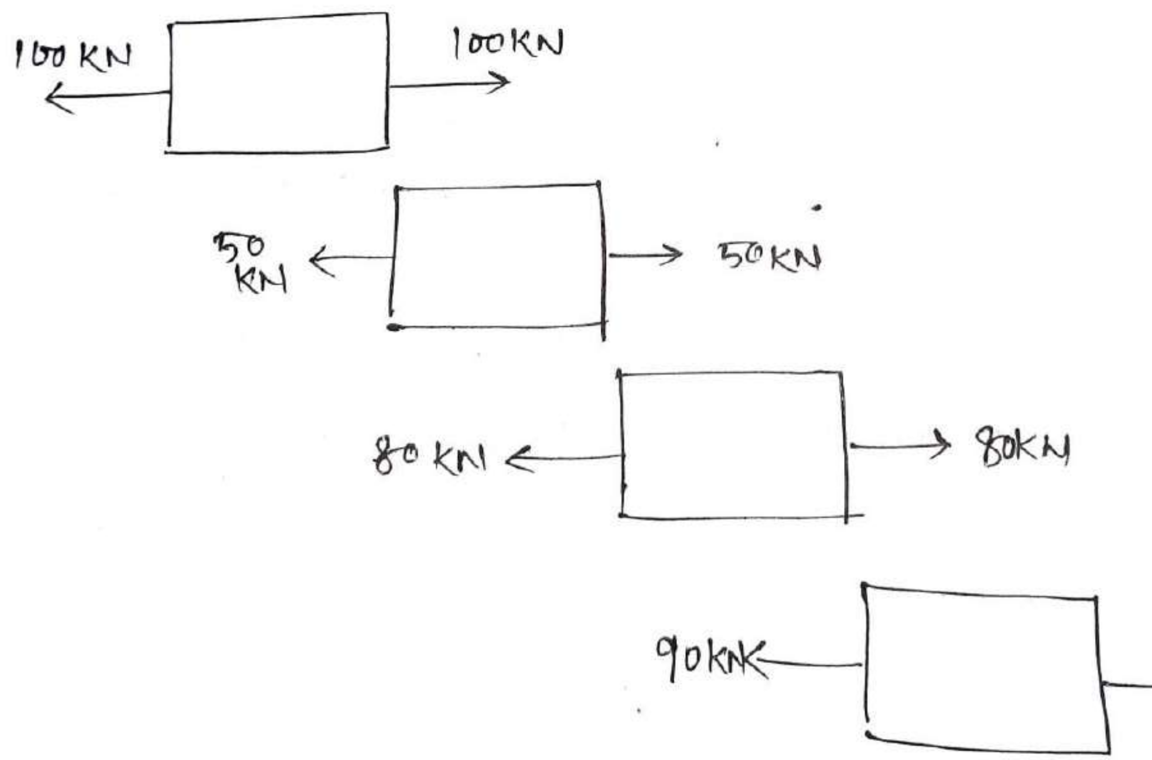
$$\Rightarrow \Delta l = 0.75mm + 1.5mm + 1mm$$

$$\Rightarrow \boxed{\Delta l = 3.25mm}$$

Q



free Body diagram:—



$A_1 = \frac{\pi}{4}(5)^2 = 19.63 m^2$

$E = 2.5\ mpa$
$= 2.5 \times 10^6$

$A_2 = \frac{\pi}{4}(4)^2 = 12.56 m^2$

$A_3 = \frac{\pi}{4}(3)^2 = 7.06\ m^2$

$A_4 = \frac{\pi}{4}(5)^3 = 19.63 m^2$

$$\Delta l_1 = \frac{P_1 L_1}{A_1 E} = \frac{100 \times 10^3 \times 1}{19.63 \times 2.5 \times 10^6} = 2.037 \times 10^{-3}\ m$$

$$\Delta l_2 = \frac{P_2 L_2}{A_2 E} = \frac{50 \times 10^3 \times 1.5}{12.56 \times 2.5 \times 10^6} = 2.38 \times 10^{-3}\ m$$

$$\Delta l_3 = \frac{P_3 L_3}{A_3 E} = \frac{80 \times 10^3 \times 1}{7.06 \times 2.5 \times 10^6} = 4.53 \times 10^{-3}\ m$$

$$\Delta l_4 = \frac{P_4 L_4}{A_4 E} = \frac{90 \times 10^3 \times 1.5}{19.63 \times 2.5 \times 10^6} = 2.75 \times 10^{-3} m$$

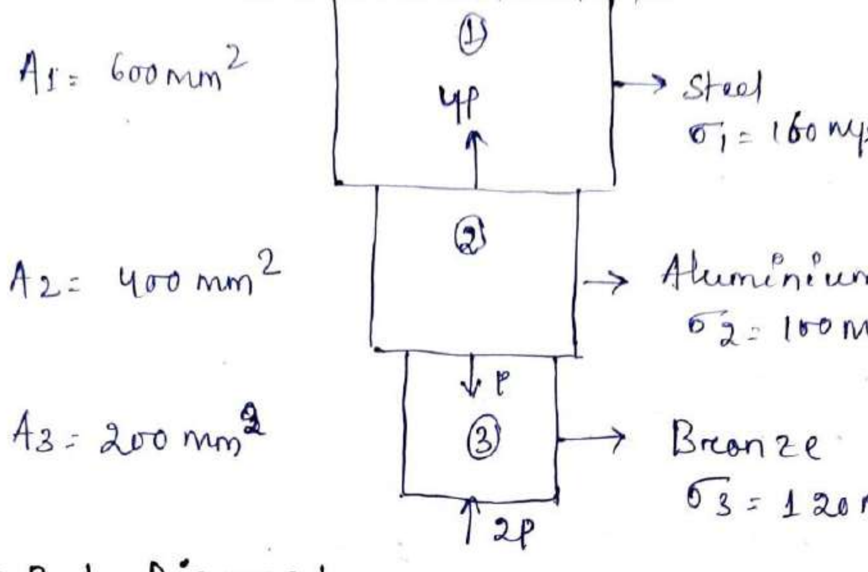
∴ Total elongation

$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4$

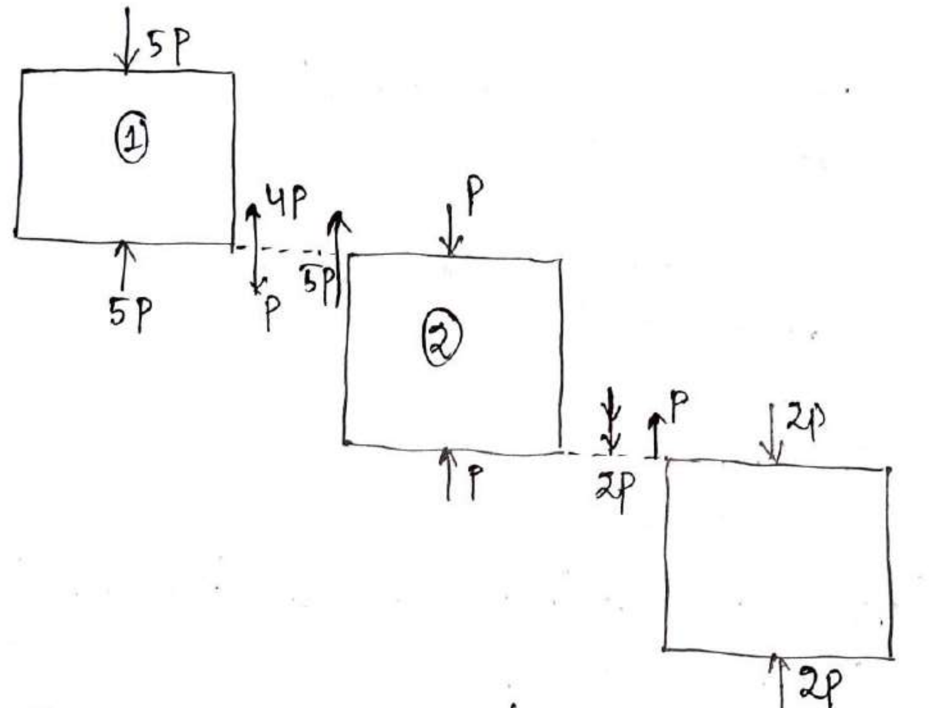
$\Rightarrow \Delta l = (2.037 \times 10^{-3}) + (2.38 \times 10^{-3}) + ($
$(2.75 \times 10^{-3})\ m$

$\Rightarrow \Delta l = 0.011697\ m$

$\Rightarrow \boxed{\Delta l = 11.697\ mm}$



Free Body Diagram :-



$\sigma_1 = 160\ MPa = 160 \times 10^6\ N/m^2 = 160 \times 10^6\ N$

$\Rightarrow \boxed{\sigma_1 = 160\ N/mm^2}$

$\sigma_2 = 100\ MPa = 100 \times 10^6\ N/m^2 = 100 \times 10^6\ N$

$\Rightarrow \boxed{\sigma_2 = 100\ N/mm^2}$

$A_3 = 200\,mm^2 \qquad P_3 = 2P$

$\therefore$ We know that:

for Section 1:-

$$\sigma_1 = \frac{P_1}{A_1}$$

$$\Rightarrow 160 = \frac{5P}{600}$$

$$\Rightarrow P_1 = \frac{160 \times 600}{5}$$

$$\Rightarrow P_1 = \boxed{19200\,N}$$

for Section 2:-

$$\sigma_2 = \frac{P_2}{A_2}$$

$$\Rightarrow 100 = \frac{P}{400}$$

$$\Rightarrow P_2 = 100 \times 400$$

$$\Rightarrow \boxed{P_2 = 40000\,N}$$

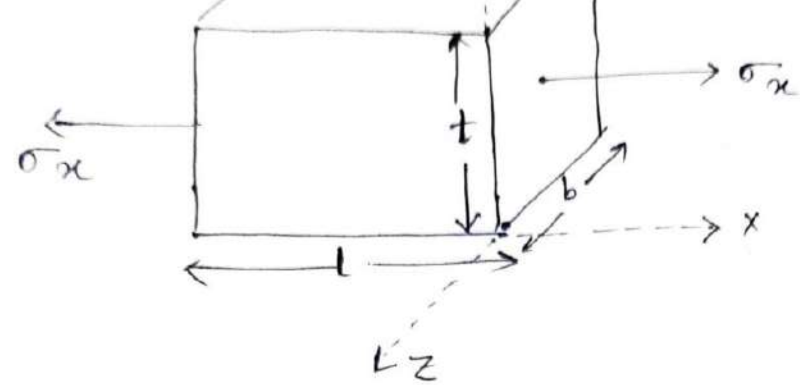
for Section 3:-

$$\sigma_3 = \frac{P_3}{A_3}$$

$$\Rightarrow 120 = \frac{2P}{200}$$

$$\Rightarrow P_3 = \frac{120 \times 200}{2}$$

$$\Rightarrow \boxed{P_3 = 12000\,N}$$



let l, b, t are the length, breadth and
the rectangular block.

σ_x = tensile stress in x-direction.

E = Young's Modulus.

μ = Poisson's ratio.

$$= \frac{-\text{lateral strain}}{\text{longitudinal strain}}$$

Longitudinal Strain :

$$\boxed{e_x = \frac{\sigma_x}{E}}$$

$$\mu = \frac{-e_y}{e_x}$$

$$\Rightarrow e_y = -\mu \cdot e_x \quad \left[\because e_x = \frac{\sigma_x}{E}\right]$$

$$\Rightarrow \boxed{e_y = -\mu \frac{\sigma_x}{E}}$$

Similarly, $\boxed{e_z = -\mu \frac{\sigma_x}{E}}$

$$\boxed{V = l \times b \times t}$$

Now, derivating both the sides:

$$\boxed{\frac{dv}{v} = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t}}$$

$$\left[\because \frac{d}{l}(l \times b \times t) + \frac{d}{b}(l \times b \times t) + \frac{d}{t} \right.$$

$$\left. = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t} \right]$$

$$\Rightarrow \boxed{ev = ex + ey + ez}$$

$$\left[\because ev = \frac{dv}{v} = \text{Volumetric Strain} \quad ex = \frac{dl}{l} \quad ez = \frac{dt}{t} \quad ey = \frac{db}{b} \right]$$

Now put the value of ex, ey and ez

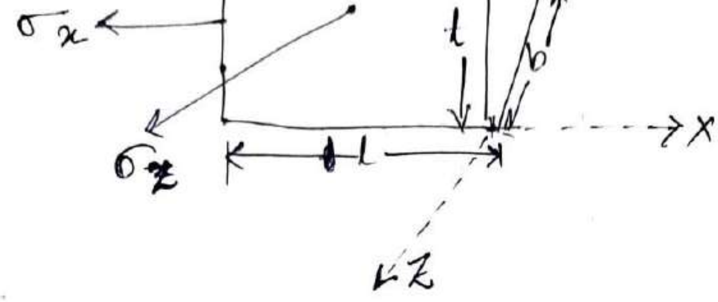
$$\boxed{ev = \frac{\sigma x}{E} - \mu \frac{\sigma x}{E} - \mu \frac{\sigma x}{E}}$$

$$\Rightarrow ev = \frac{\sigma x}{E} - 2\mu \frac{\sigma x}{E}$$

$$\Rightarrow \boxed{ev = \frac{\sigma x}{E}(1 - 2\mu)}$$

(2) Volumetric Strain of a rectangular block s
three mutually perpendicular Stress:

Let l, b, t are the length, breadth and th
rectangular block.
$\sigma x, \sigma y, \sigma z$ are the three stresses act in t
of x, y, z.



We know that, $V = l \times b \times t$

and $\frac{dv}{v} = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t}$

$\Rightarrow \boxed{e_v = e_x + e_y + e_z}$

- $e_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \frac{\mu\sigma_z}{E}$
- $e_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \mu\frac{\sigma_z}{E}$
- $e_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$.

$\therefore e_v = e_x + e_y + e_z$

$= \left(\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}\right) + \left(\frac{\sigma_y}{E} - \mu\right.$

$\left(\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}\right)$

$\Rightarrow e_v = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} -$

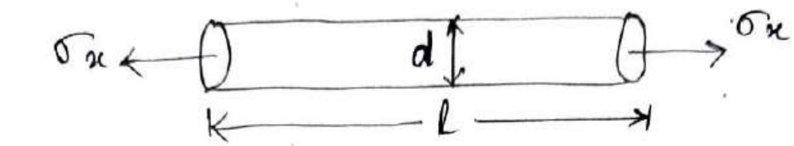
$\mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$.

$\Rightarrow e_v = \frac{1}{E}\left[\sigma_x - \mu\sigma_y - \mu\sigma_z + \sigma_y - \sigma_x - \mu\sigma_z -\right.$

$\Rightarrow e_v = \frac{1}{E}\left[\sigma_x - \mu\sigma_x - \mu\sigma_x + \sigma_y - \mu\sigma_y - \mu\sigma\right.$

$\left.\mu\sigma_z\right]$

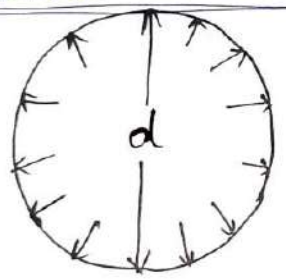
(3) Volumetric Strain for a Circular Rod:



$$\boxed{e_v = 2e_d + e_l}$$

where, e_v = Volumetric Strain
e_d = lateral Strain
e_l = longitudinal strain.

(4) Volumetric Strain for a sphere:



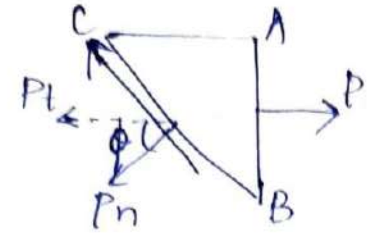
$$\boxed{e_v = 3e_d}$$

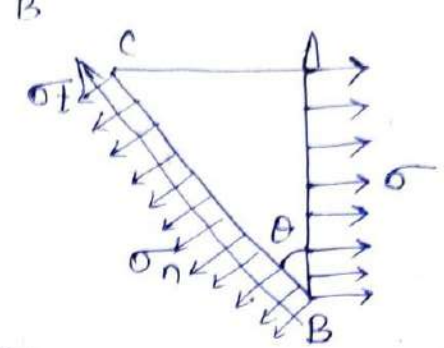
Mo

Complex stress and strain:

Principal stresses and strains:

- Principal planes are these planes within th
such that the resultant Stresses across
wholly normal stresses or planes across c
shearing Stresses occur.
- Principal stresses are those stresses w
on the principal planes (plane on which there
- The plane carrying the maximum norm
called the major principal plane and t
acting on it is called major princi
- the plane carrying minimum normal stress i
minor principal plane and the stress acti
called as minor principal stress.





Normal component of force, $P_n = P \cdot \cos\theta$

Tangential component of force, $P_t = P \cdot \sin\theta$

Normal stress, $\sigma_n = \sigma \cdot \cos^2\theta$

Tangential stress, $\sigma_t = \sigma \cdot \frac{\sin 2\theta}{2}$

P = Applied tensile force.

σ = Stress.

Resultant stress, $\sigma_r = \sqrt{\sigma_n^2 + \sigma_t^2}$
r = resultant

ϕ = Angle of the resultant stress with
stress.

$\tan\phi = \frac{\sigma_t}{\sigma_n}$

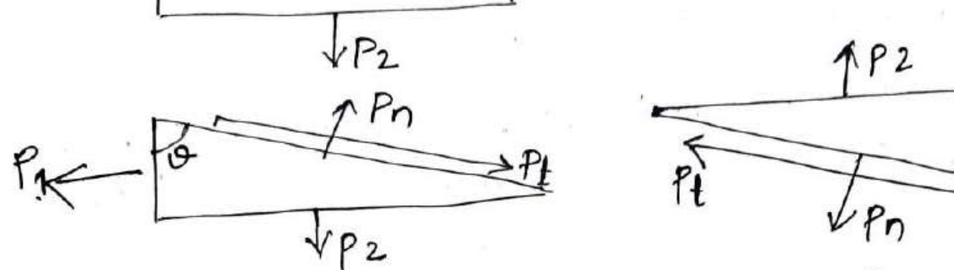
σ_n is maximum when $\cos^2\theta = 1 \Rightarrow \theta = 0°$

Maximum normal stress, $(\sigma_n)_{max} = \sigma$

σ_t is maximum when $\sin 2\theta = \pm 1 \Rightarrow 2$

$\theta = 45°$, or $135°$.

Maximum tangential stress, $(\sigma_t)_{max} =$



Normal stress, $\sigma_n = \sigma_1 \cdot \cos^2\theta + \sigma_2 \sin^2\theta$

$$\sigma_n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2} \cdot \cos 2$$

Tangential stress. $\sigma_t = \frac{\sigma_1-\sigma_2}{2} \sin 2\theta$.

σ_t is maximum when $\sin 2\theta = \pm 1$

$\Rightarrow 2\theta = 90^\circ$ or 270°,

$\theta = 45^\circ$ or 135°.

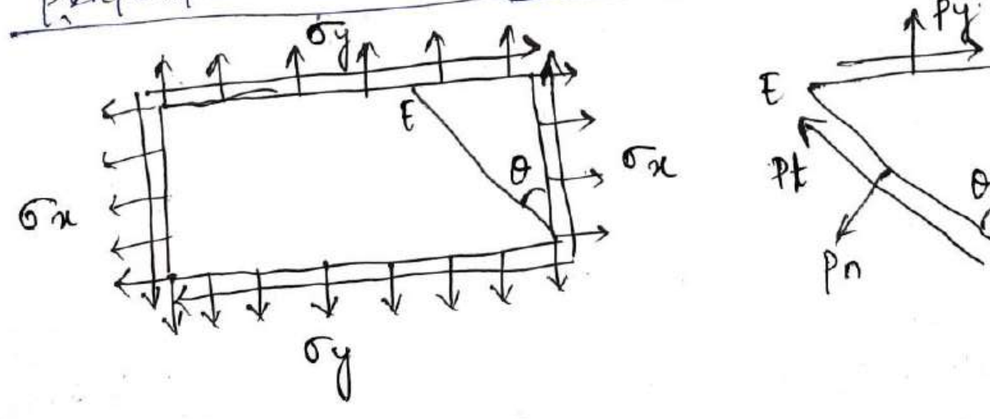
Maximum tangential stress $(\sigma_t)_{max} = \pm$

Resultant stress, $\sigma_r = \sqrt{\sigma_n^2 + \sigma_t^2}$

ϕ = Angle of the resultant Stress with

$$\tan\phi = \frac{\sigma_t}{\sigma_n}$$

Stresses on an inclined plane Subjected
Perpendicular normal stresses and She



Since $\tan(180+2\theta) = \tan 2\theta$, there are ...
Satisfying the above relation. The valu...
by 90°.

The principal Stresses are;

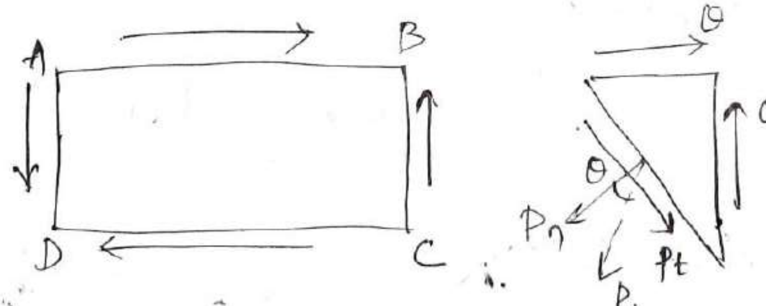
$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2 \ldots}$$

The principal Stresses may like or unlike. ...
maximum shear Stress will be inclined ...
$(\theta+135^\circ)$ to the plane BC.

Maximum shear Stress,

$$\tau_{c\,max} = \frac{\sigma_1-\sigma_2}{2} = \sqrt{\frac{(\sigma_x-\sigma_y)^2+4\ldots}{2}}$$

Stresses on an inclined plane Subjected ...



Normal Stress, $\sigma_n = \tau \cdot \sin 2\theta$.

Tangential Stress, $\sigma_t = \tau \cos 2\theta$

Resultant Stress, $\sigma_r = \sqrt{\sigma_n^2+\sigma_t^2}$

ϕ : Angle of the resultant stress with ...
Stress.

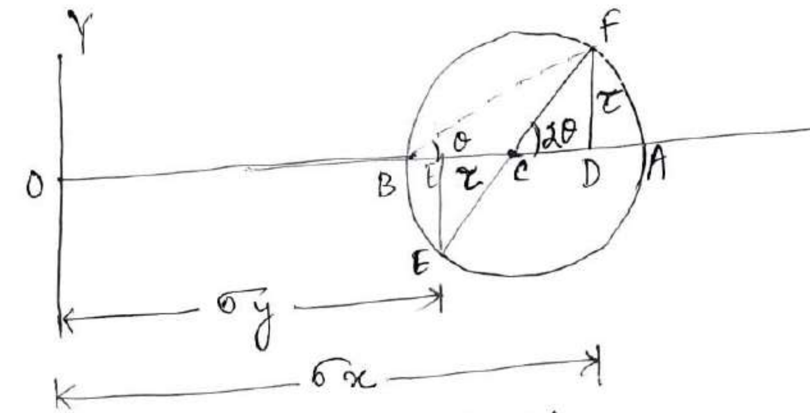
The planes inclined at 45° ... Stress.

At $\theta = 45°$, tangential Stress, $(\sigma_n)_{\theta - 45°} =$

At $\theta = 135°$, tangential Stress, $(\sigma_n)_{\theta - 135°} =$

Mohr's Circle Method

1. for finding principal stresses :—



Major principal Stress, $\sigma_1 = OA$

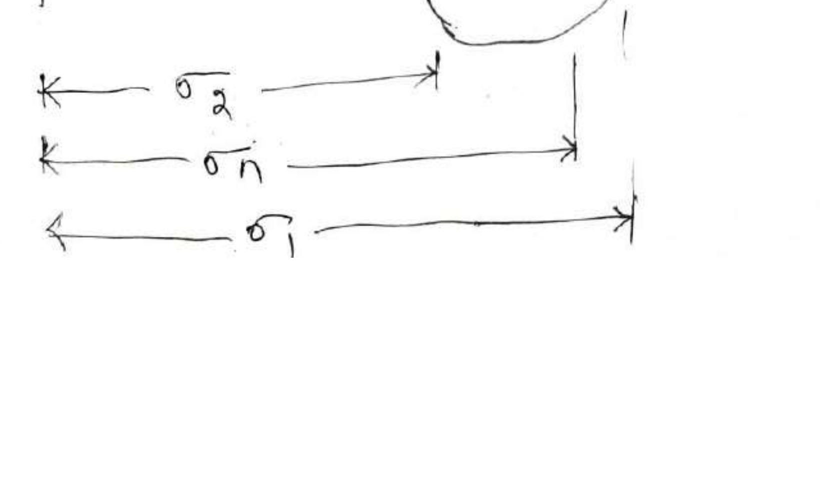
Minor principal Stress, $\sigma_2 = OB$

The radius of the circle represents the maxim...

θ = Inclination of one of the principal planes ...

plane normal to which the major stre...

2. for finding normal stress :—



the Mohr's circle.

$$CF' = \frac{\sigma_1 - \sigma_2}{2}, \quad OC = \frac{\sigma_1 + \sigma_2}{2}$$

Maximum angle of the resultant st
normal stress (ϕ_{max}) is given by s

$$= \frac{CF'}{OC} = \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}.$$

Strains on an inclined plane;

let ε_x: strain in x direction

ε_y: strain in y direction

ϕ_{xy}: shear strain on xy plane

ε_θ: strain an inclined plane
of θ with the major pri

ϕ_θ: shearing strain on a p
at angle of θ :-

$$\varepsilon_\theta = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cdot \cos 2\theta + \phi$$

$$\frac{\phi_\theta}{2} = \frac{e_x + e_y}{2} \sin 2\theta - \frac{\phi_{xy}}{2} \cdot Co$$

Principal strains in two dimens

$$\varepsilon_1, \varepsilon_2 = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 +}$$

$$\tan 2\theta = \frac{\phi_{xy}}{\varepsilon_1 - \varepsilon_2}$$

$\varepsilon_3 = -\frac{1}{m}\cdot\frac{\sigma_1}{E} - \frac{1}{m}\cdot\frac{\sigma_2}{E} + \frac{\sigma_3}{E}$.

for 2D system,

$\phi_{max} = \varepsilon_1 - \varepsilon_2$

The maximum shear strain is equal to of principal strains.

Strain energy due to principal stresses:

Strain energy, $U = \frac{1}{2}\cdot\sigma_1\cdot\varepsilon_1 + \frac{1}{2}\sigma_2\cdot\varepsilon_2$

$= \frac{1}{2E}\left(\sigma_1^2+\sigma_2^2+\sigma_3^2\right) - \frac{1}{mE}\left(\sigma_1\sigma_2\right.$

for 2D system, $\sigma_3 = 0$

$$U = \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2 - \frac{2}{mE}\sigma_1\sigma_2\right]$$

Computation of principal stresses from Strain :—

$$\sigma_1 = E\left[\frac{(1-\mu)\varepsilon_1 + \mu(\varepsilon_2+\varepsilon_3)}{(1+\mu)(1-2\mu)}\right]$$

$$\sigma_2 = E\left[\frac{(1-\mu)\varepsilon_2 + \mu(\varepsilon_2+\varepsilon_1)}{(1+\mu)(1-2\mu)}\right]$$

$$\sigma_3 = E\left[\frac{(1-\mu)\varepsilon_3 + \mu(\varepsilon_1+\varepsilon_2)}{(1+\mu)(1-2\mu)}\right]$$

for 2D stress system, $\sigma_3 = 0$

$$\sigma_1 = E\cdot\left[\frac{\varepsilon_1 + \mu\cdot\varepsilon_2}{1-\mu^2}\right]$$

$$\sigma_2 = E\left[\frac{\mu\cdot\varepsilon_1 + \varepsilon_2}{1+\mu^2}\right]$$

... in perpendicular di
Constant.

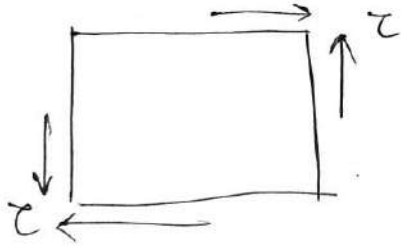
$$\varepsilon_{\theta_1} + \varepsilon_{\theta_2} = \varepsilon_1 + \varepsilon_2$$

The maximum shear strain in xy plane with axis at 45° to the direction of planes.

$$\frac{\phi_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

Q If an element of a stressed body is in a st shear with a magnitude of 80 N/mm², of maximum principal stress at that l

Ans



Shear stress : $\tau = 80\ N/mm^2$

Normal stress in x direction, $\sigma_x = 0$

Normal stress in y direction, $\sigma_y =$

Maximum principal stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)}$$

$$\sigma_1 = 80\ N/mm^2$$

Q The state of 2D stress acting on a conc consists of a direction tensile stress, σ and shear stress $\tau = 1.20\ N/mm^2$, which of ~~force~~ concrete. Then what is the tensile st concrete in N/mm²?

The major and minor principal stresses are giv

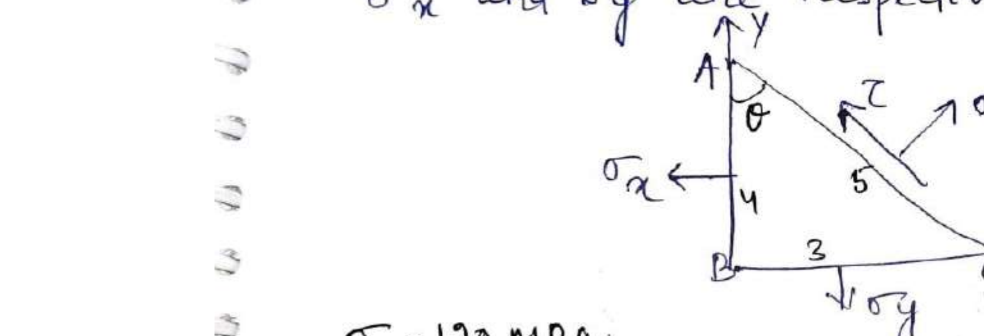
$$\sigma_{1,3} = \frac{\sigma_x+\sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau$$

$$= \frac{1.5}{2} + \frac{1}{2}\sqrt{(1.5)^2 + 4(1.2)^2} = 0.75 \pm \frac{1}{2}$$

$$= 0.75 \pm 1.$$

$$\sigma_1 = 0.75 + 1.42 = 2.42 = 2.17 \text{ N/}$$

Q In a 2D stress analysis, the state of str
is shown below. If $\sigma = 120$ mpa and $\tau = 7$
σ_x and σ_y are respectively.



well conditioned

$\sigma = 120$ mpa.
$\tau = 70$ mpa.

$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = 3/4$

Considering the horizontal equilibrium,

$$\boxed{\sigma_x \times AB = AC\,(\sigma\cos\theta - \tau\sin}$$

$$\sigma_x \times 4 = 5\left(120 \times \frac{4}{5} - 70 \times \frac{3}{5}\right)$$

$\Rightarrow \sigma_x = 67.5$ Mpa.

Considering vertical equilibrium,

$$\sigma_y \times BC = AC\,(\sigma\sin\theta - \tau\cos$$

$$\Rightarrow \sigma_y \times 3 = 5\left(120 \times \frac{3}{5} + 70 \times \frac{4}{}\right.$$

$\Rightarrow \sigma_y = 213.3$ Mpa,

: 15 Mpa.

Mod

03 Stresses in Beams and shafts:–

Stress in Beams due to Bending: or Bending

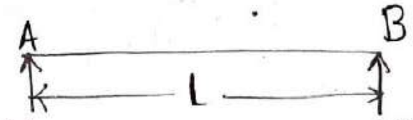
When a beam is loaded with external the Sections of the beam will experience moments and Shear forces.

Shear force is defined as the algebraic the forces acting on either Side of the

Bending moment is defined as the forces either Side of the section.

Types of Beams:

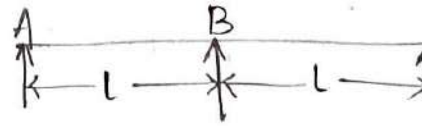
Simply Supported Beam



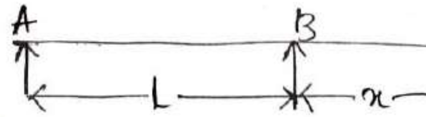
fixed Beam

Cantilever Beam

Continuous Beam



Over hanging Beam



Simple Bending or Pure Bending:

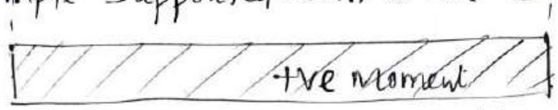
A beam or a part of it is Said to be of pure bending when it bends under the uniform or Constant bending moment, withou

Normal Stress:

However, in practice, when a beam is subject[...] loads, the bending moment at a Section is a[...] by shear force. But, it is generally observe[...] Shear force is Zero where the bending moment [...]

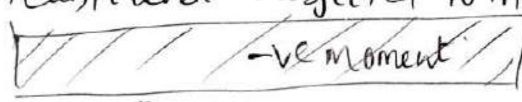
Examples of pure Bending:

(a) Simple Supported beam with ~~end~~ end Coupling

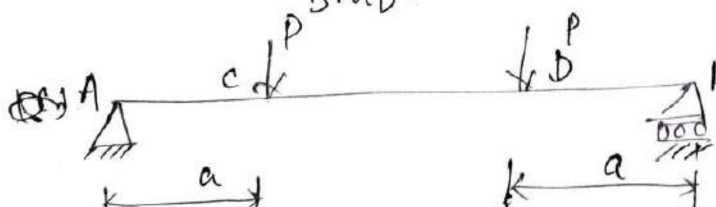


Bending Moment diagram

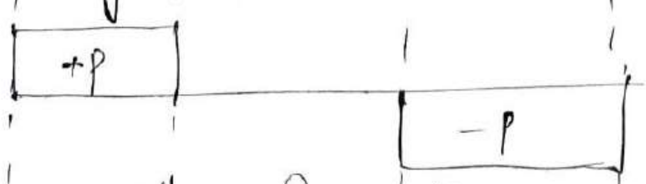
(b) Cantilever Subjected to moment at [...]



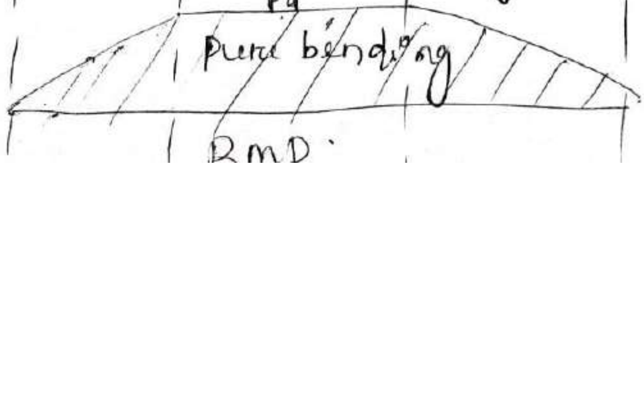
BMD.



(C) Simply Supported Beam with two-poi[...]



Shear force diagram.



BMD.

- In this chapter, bending of straight ... uniform cross-sectional area with vertical ... Symmetry shall be considered.
- The application of this theory can be extend... with two or more different material as ... curved beams.

Neutral Surface :-

Bending moment causes the material fibr... bottom portion of the beam to stretch and ... fibre in the top portion to compress.

Consequently, between these two regions the... one such surface called neutral surface ... longitudinal fibres of the material will no... any change in length or remain free ... kind of bending stress.

Neutral axis : NA

Line formed by intersection of neutral sur... corresponding plane of cross-section is call... axis.

If the section of the beam is symmetric an... is isotropic and homogeneous, then the ... axis passes through the geometric centroid ... cross-section.

It is the axis where strain changes i...

Cross-section of the Beam.

Tension

Strain distribut

Equation of pure Bending:

Assumptions:

Assumptions taken while deducting pure ben

→ Beam is initially straight and has a ~~cross~~ cross-section.

→ plane Cross Sections before bending remain pl (Bernoulli's Assumption).

→ Beam material is homogeneous, isotropic a Law and limits of eccentricity are not ex

→ Every layer is free to expand or contr

→ Modulus of elasticity of the beam mat value in tension and Compression.

→ The beam is subjected to pure bendin bends into an arc of a circle.

→ Radius of curvature is large compared of the cross-section.

Bending equation :-

$$\boxed{\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}}$$

where, M = Mome

I = Moment of In Section about

E = Young's Mod

R = Radius of curvature of Neutral axis.

σ = Bending stress.

Y = Distance from the NA to the extreme fi

Sol: Given;

Diameter of given wire, d = 20mm.

Radius of curvature, R = 1m = 10,000 mm

Modulus of elasticity, E = 2×10^5 N/mm^2.

from Bending equation;

$$\frac{E}{R} = \frac{\sigma}{y}$$

$$\Rightarrow \frac{2\times10^5}{10,000} = \frac{\sigma}{\left(\frac{20}{2}\right)}$$

$\Rightarrow$ Maximum Bending stress, σ = 20

Moment of Resistance (M_r):

It is the maximum Bending moment which carried by a given section for a given m value of stress.

Flexural ~~rigity~~ rigidity :-

Product of Young's modulus (E) and mome inertia (I) of the cross-section of the beam flexural rigidity (EI).

* flexural rigidity is a parameter which r flexural stiffness of the beam.
* Bending equation is also called flexura

Necessity of flexural formulae:

- Bernoulli's assumption gives linear var strain upto failure point.

a. Deep beams

b. Torsion on non circular members.

- The strain of a fibre is proportional to it
  NA, $\varepsilon = \frac{Y}{R}$.

- the Compressive forces above Neutral la
  equal to the tensile forces below neutra

Section Modulus:

For simple bending, Maximum bending st
Cross-section is given by.

$$\boxed{\sigma_{max} = \frac{M}{I} \times Y_{max}}$$

$$\therefore \boxed{M = \frac{I}{Y_{max}} \times \sigma_{max}}$$

$$= Z \times \sigma_{max}$$

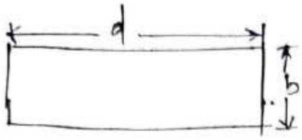
Here, the ratio $\frac{I}{Y_{max}}$ is denoted by 'Z'
section modulus of the cross-section.

Significance:

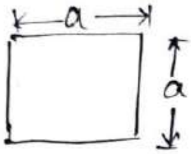
It represents the strength of the
Greater the value of 'Z', stronger
cross-section of beam against bendi

Note: The higher the value of Section m
particular section the higher the ben
which it can withstand for a given M

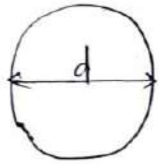
$\frac{a}{2}$ $\quad 6$



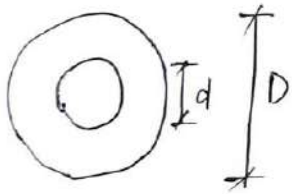
$$\frac{\frac{db^3}{12}}{\frac{b}{2}} = \frac{db^2}{6}$$



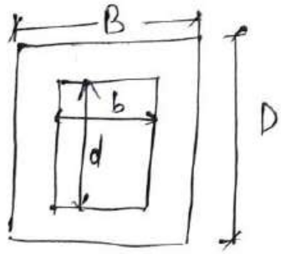
$$\frac{\frac{a\cdot a^3}{12}}{\frac{a}{2}} = \frac{a^3}{6}$$



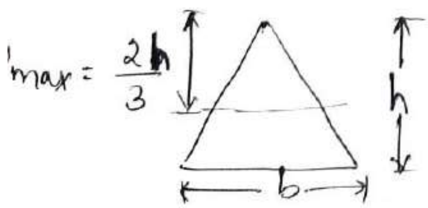
$$\frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^3}{32}$$



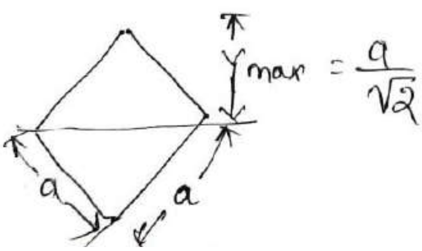
$$\frac{\left(\frac{\pi}{64} D^4 - d^4\right)}{\frac{D}{2}} =$$



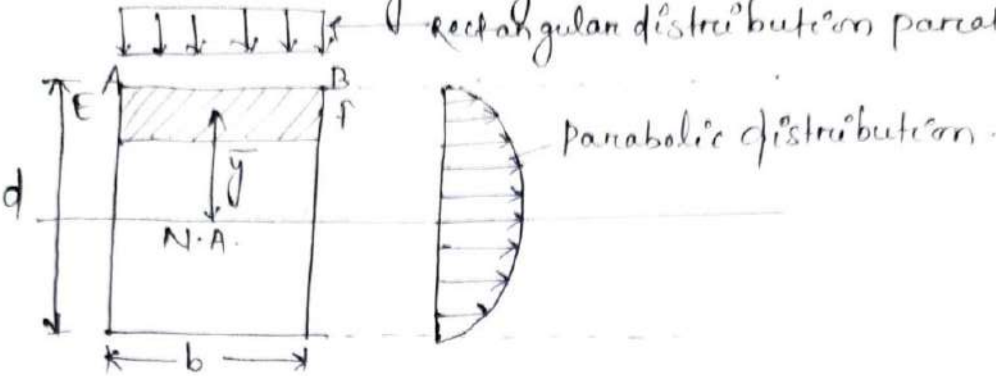
$$\frac{\frac{BD^3 - bd^3}{12}}{\frac{D}{2}} = BD$$



$$\frac{\frac{bh^3}{36}}{\frac{2h}{3}} =$$



$$\frac{\frac{a\cdot a^3}{12}}{\frac{a}{\sqrt{2}}} =$$



$$\boxed{\tau = \frac{FA\bar{y}}{Ib}}$$

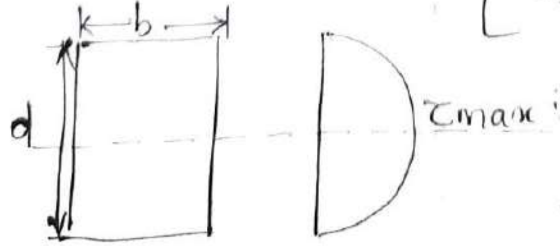
where; F = Shear force at a given cross-section
A = Area either above or below the Sect
$\bar{y}$ = Distance of centroid of an area 'A
b = width of the beam section.
I = Moment of inertia of entire cro about NA.

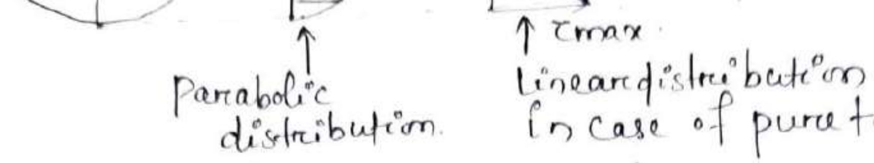
Shear stress distribution for beam section of Shapes:—

a. Rectangular Section:

Average Shear stress; $\tau_{avg} = (F/bd)$

$[\tau_{max} = (3/2)\tau_{av}$





(Shear stress distribution)

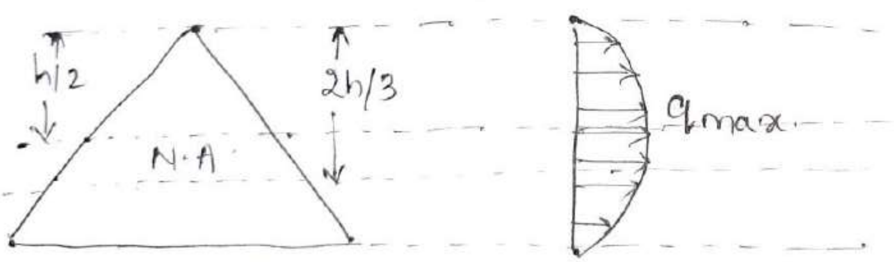
$$\left[\tau_{max} = (4/3)\,\tau_{avg}\right]$$

c) <u>For triangular Section:-</u>

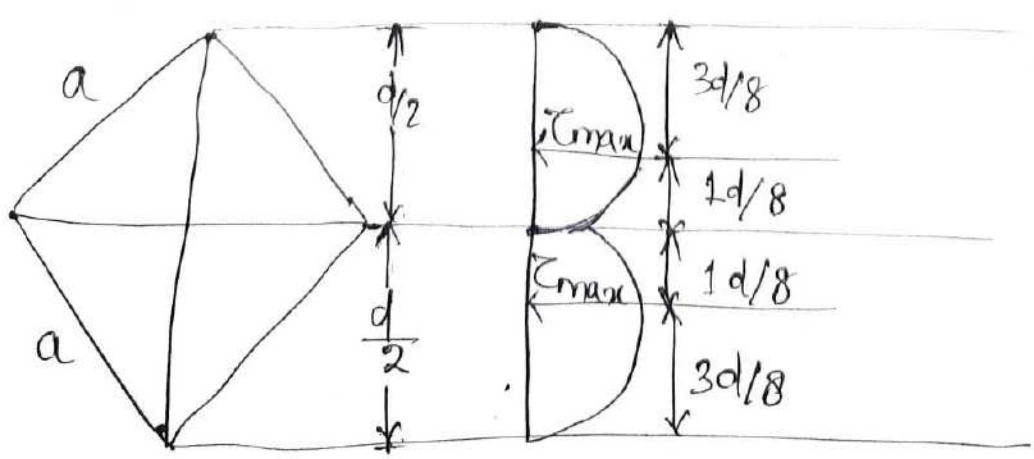
$$\tau_{max} = 3\,(F/bh)$$
$$= (3/2)\,\tau_{avg}$$

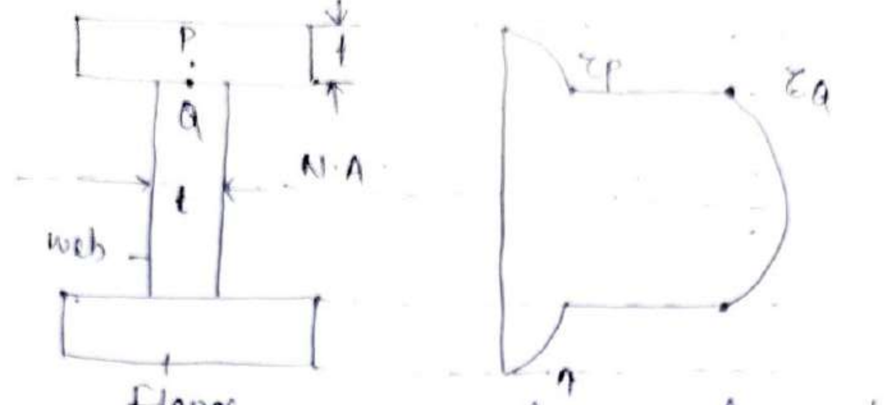
at h/2 from apex or base.

$$\tau \text{ at N.A.} = (8F/3bh) = (4/3)\,\tau_{avg}$$



d) A beam of Square Section is placed diagonal placed horizontally.





Average shear st

Section	τ_{max}/τ_{avg}	τ_{NA}/τ_a
Rectangle or Square	3/2.	3/2
Solid Circular	4/3	4/3
Triangle	3/2	4/3
Diamond	9/8	1

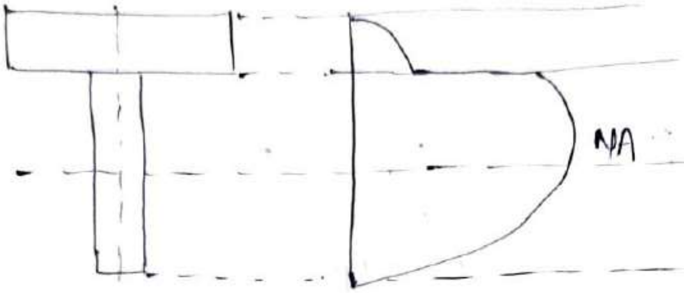
Q A timber beam is 100 mm wide and 150 mm d is simply supported and carries a central load W. If the maximum stress in shear what would be the corresponding load W

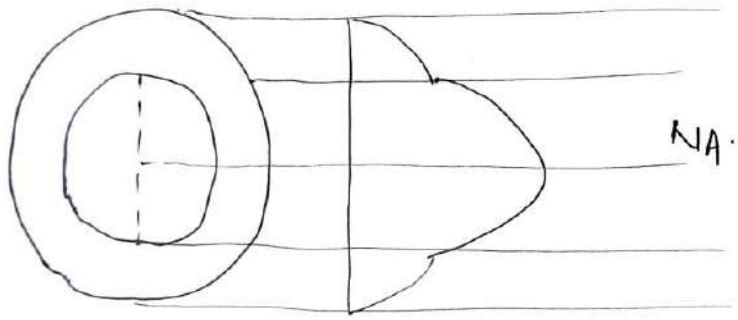
Ans for a rectangular cross section

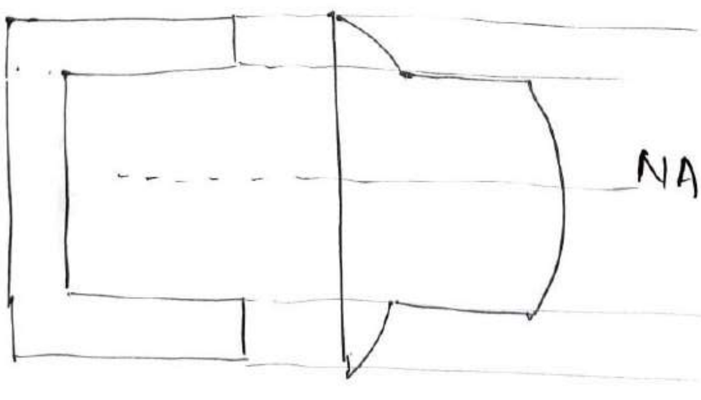
$$\tau_{max} = \frac{3}{2}\left(\tau_{avg}\right)$$

$$\Rightarrow 2 = \frac{3}{2}\left[\frac{W}{100 \times 150}\right]$$

$$\Rightarrow W = 40\,KN. \quad \text{Ans}$$


NA


NA·


NA·

Torque - Rotational equivalent of linear fo

Assumptions:

1. Material of shaft taken is homogeno
2. Stresses are within elastic limit So
   proportional to strain.
3. Plane normal sections of shaft rem
   twisting.
4. Radii remain straight after torsion

Formula

$$\boxed{T = \frac{J_T}{r}\,\tau}$$

T = Applied torque or moment of tors

τ = Maximum shear stress at the

J_T : Torsion constant for the secti

r : Rotational axis and the farth
    section. (At the outer surface

Torsional / Polar section modulus (Z_p)

$$\boxed{Z_p = \frac{J}{r_{max}}}$$

J = polar moment of inertia of c
    shaft about longitudinal a

r_{max} : Radius of shaft section.

$$\frac{\quad 32}{d/2}$$

$$\Rightarrow \boxed{Z_p = \frac{\pi d^3}{16}}$$

- For a hollow circular shaft.

$$Z_p = \frac{\frac{\pi}{32}\{D^4 - d^4\}}{D/2}$$

$$= \frac{\pi}{16}\frac{\{D^4 - d^4\}}{D}$$

$$= \frac{\pi}{16} D^3 \left\{1 - \left(\frac{d}{D}\right)^4\right\}$$

$$\boxed{Z_p = \frac{\pi}{16} D^3 \{1 - K^4\}}$$

where, $\cancel{\frac{\pi}{16}}$ $K = d/D$.

Torsional Rigidity :—

Torque required to produce unit angula

$$\boxed{GJ = \frac{Tl}{\theta}}$$

GJ = Torsional rigidity

T = Torque applied.

l = Length of shaft.

θ = Twist of the cross-section.

Unit: N-m/radian.

Unit N-m/radian.

Angle of twist:

Angular deflection of longitudinal fibres give

$$\boxed{\theta = \frac{Tl}{GJ}}$$

Unit: radians.

Power Transmitted by a shaft:

$$\boxed{P = \frac{2\pi NT}{60}}$$

where, T = Average torque in N-m.

N = Shaft speed rpm.

Q find the power transmitted by a circular
50mm diameter at 120 rpm. the maximum s
in the shaft is not to exceed 60 N/mm^2.

Soln Given; Diameter of the shaft = d = 50 mm

Speed, N = 120 rpm.

Maximum shear stress τ = 60 N/

Power, P = ?

$$P = \frac{2\pi NT}{60} \text{ watts}$$

$$\boxed{T = \frac{\pi}{16} \times \tau \times d^3}$$

Torsion equation;

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}}$$

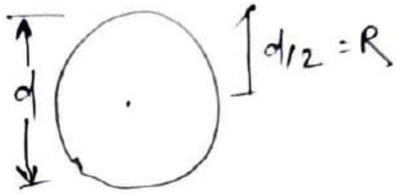
(Radius).

G = Modulus of rigidity

θ = Angle of twist.

L = Length of the shaft.

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$$



$$\frac{T}{J} = \frac{\tau}{R}$$

$$\Rightarrow T = \frac{J}{R} \cdot \tau$$

$$= \frac{\frac{\pi d^4}{32} \times \tau}{d/2}$$

J = Polar MI.

$$J = \frac{\pi}{32} d^4$$

$$\Rightarrow \boxed{T = \frac{\pi}{16} \times \tau \times d^3}$$

$$T = \frac{\pi}{16} \times 60 \times 50^3$$

$$= 1472621.556 \text{ Nmm}$$

$$T = 1472.621 \text{ Nm}$$

$$P = \frac{2\pi NT}{60} = \frac{2 \times \pi \times 120 \times 1472.621}{60}$$

$$= 18505.50 \text{ N}$$

$$\Rightarrow P = 18.50 \text{ KN}.$$

torque is applied ?

Given;

outside diameter, $D = 120 mm$

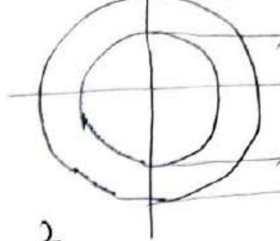
Inside diameter, $d = 90 mm$

Allowable shear stress, $\tau = 60 N/mm^2$

find (i) Torque $T = ?$

(ii) stress at inner surface = ?

i.e at $r = 45 mm$.



Torque transmitted by a hollow shaft;

(i)
$$\boxed{T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right)}$$

$$= \frac{\pi}{16} \times 60 \times \left(\frac{120^4 - 90^4}{120}\right)$$

$$= 13916273.7078 \text{ N·mm}$$

$$T = 13916.27 \text{ N·m}.$$

(ii) Stress at the inner Surface:

i.e $r = 45 mm$.

$$\tau \propto R \; ; \; \frac{\tau}{R} = C \rightarrow \frac{\tau_1}{R_1} = \frac{\tau_2}{R_2}$$

[Lets take $\tau_1 = \tau$, $R_1 = R$.

$$\frac{\tau}{R} = \frac{q}{r}$$

$$\Rightarrow q = \frac{\tau}{R} \cdot r$$

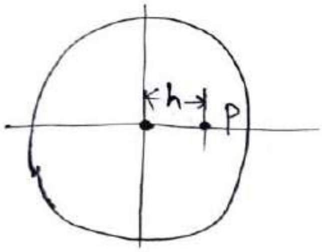
$$\Rightarrow q = \frac{60}{60} \times 45 = 45 N/mm^2.$$

of action of eccentric
Load.

$\sigma_c = \frac{P}{A}$
(Bending s

$\sigma_b =$

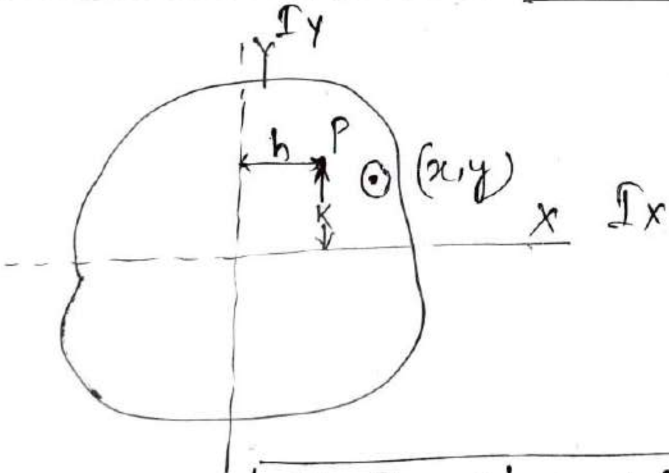
$\frac{\sigma_b}{y} = \frac{M}{I} = \frac{E}{R}$ $\sigma_b =$



Totalstress = σ = Direc
stress + Bending

$$\boxed{\sigma = \frac{P}{A} + \frac{Phy}{I}}$$

<u>Load is Eccentric about Both the axis</u>

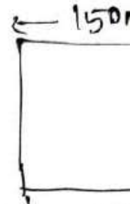


$$\boxed{\sigma = \frac{P}{A} + \frac{Phx}{I_y} + \frac{Pky}{I_x}}$$

A rectangular strut is 150mm and 120m
t carries a load of 180 kN at an
f 10mm in a plane bisecting the thickness
aximum and minimum intensities of s
section.

$\sigma_{max} = ?$ 										ele…

$\sigma_{min} = ?$



$$\boxed{\sigma_{max} = \sigma_d + \sigma_b}$$

σ_d = direct stress

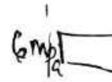
σ_b = Bending stress. $\left(\frac{M}{I} = \frac{\sigma}{y}\right) = \frac{M}{Z} = \sigma_b$. Pl…

(∵ Z…

$$\sigma_{max} = \frac{P}{A} + \frac{M}{Z} \rightarrow (P \times e)$$

$$= \frac{180 \times 10^3}{150 \times 120} + \frac{180 \times 10^3 \times 10}{\frac{120 \times 150^3}{12 \times 150/2}}$$

$\sigma_{max} = 14 \text{ mpa}$.



$$\boxed{\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)}$$

$$\boxed{\sigma_{min} = \frac{P}{A} - \frac{M}{Z}} \quad \boxed{\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)}$$

→ for … str…

$\sigma_{min} = 6 \text{ mpa}$.

Q. A rectangular column 200mm wide and 15… Carrying a vertical load of 120 KN at a… of 50mm in a plane bisecting the thickness… the maximum and minimum intensities… in the section.

Sol^n Given; $b = 200\text{mm}$

$d = 150\text{mm}$

$P = 120$

$e = 50$

$\sigma_{max} = 10 \text{ mpa}$.

min intensity

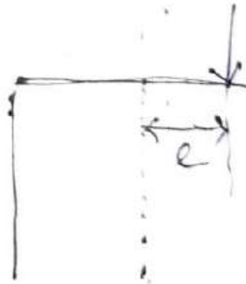
$\sigma_{min} = \sigma_d - \sigma_b = \frac{P}{A} - \frac{M}{Z}$ — Pe Imp. — I/y

$$\boxed{\sigma_{min} = \frac{P}{A}\left(1 - \frac{\sigma e}{b}\right)}$$

— x —

$\sigma_{min} = 2 \text{ mpa}$

Q In a tension specimen 13 mm in diameter the l… pull is parallel to the axis of the specimen but … displaced from it. Determine the distance of the l… pull from the axis, when the maximum ~~shear~~ s… 15% greater than the mean stress on a section … to the axis.

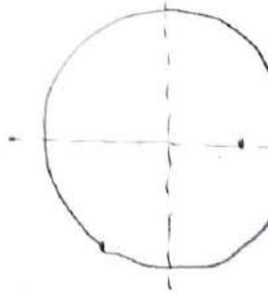


$e = ?$

$\sigma_{max} = 15\%$ greater σ ~~mean~~ mean.

$\sigma_{max} = \sigma_d + \sigma_b$

$\Rightarrow \sigma_{max} = \frac{P}{A} + \frac{M}{Z} = \frac{P}{A} + \frac{P \cdot e}{I/y}$



$\sigma_{max} = \frac{P}{A} + \frac{P \cdot e}{\frac{\pi d^3}{32}}$

$$\boxed{I = \frac{\pi}{64} d^4 \quad y = \frac{d}{2}}$$

$\sigma_{max} = 15\%$ greater. σ_{mean}

$\boxed{\sigma_{mean} = \frac{P}{A} \Rightarrow \sigma_{max} = \frac{115}{100}\frac{P}{A}}$

$\frac{115}{100}\frac{P}{A} = \frac{P}{A}$

$Z = \frac{\pi \cdot d^{\cancel{4}\,3} \cdot 2}{\underset{32}{\cancel{64}} \cancel{d}}$

$$\frac{115}{100} = 1 + \frac{8e}{d}$$

$\Rightarrow e = 0.25$ mm.

* Problems

Module: 04

04 Columns and struts:-

Introduction

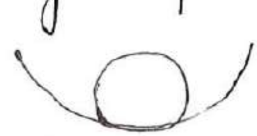
→ A bar or member of a structure (stable sy... position acted upon by a compressive load ... a strut.

→ When a compressive load is in a vertical posi... experiencing member is called as column.

→ A column can be classified as short column ... long column depending upon its failure m...

- Equilibrium of a column may be of three t...

(i) Stable equillibrium



(ii) Neutral equilibrium.

(iii) Unstable equilibrium.

* If a small axial load is applied to a colum... deformed and the column returns to its o... position after the removal of load, then ... to be in stable equilibrium.

If the load is equal to ultimate (short c... buckling load (long column) then the col... to be in neutral equilibrium. *Buckling -

where, σ_c = Ultimate crushing stress.

Long Column:

The resistance of a member to bending is due to flexural rigidity EI.

Also
Radius of gyration or radius of inertia (K) of is given by.

$$K = \sqrt{\frac{I}{A}}$$

$$\boxed{I = K^2 A.}$$

where; I = Moment of Inertia of the cross-section
A = Area of cross-section.

NOTE Resistance of a member to bending is pr moment of inertia of cross-section.

Buckling load / Crippling load / Critical loa

The load at which Column starts buckling is called buckling load.

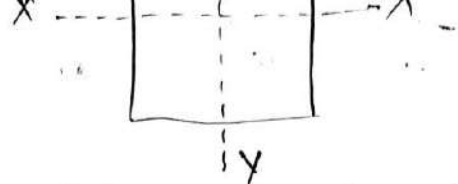
Slenderness Ratio:– (λ)

It is defined by ratio of effective length column to the least radius of gyration section.

$$\boxed{\lambda = \frac{l_e}{K_{min}}}$$

—x—

As slenderness ratio increases, permissi critical stress reduces, Consequently, load Capacity also reduces.



$I_{min} = I_y \Rightarrow$ Column buckles about Y-axis.

→ For a given area, tubular section will have
radius of gyration.

→ H-section is more efficient that I-section.

—x—

Euler's theory: (not in Diploma syllabus)

Assumption:

- Column is initially perfectly straight and axia
- Section of column is uniform.
- The material is perfectly elastic, homogeneo
  obeys Hook's law.
- Length of column is very large compared
  dimension.
- Direct stress is small compared to bending s
  to buckling condition.
- Self weight of column is ignorable.
- The column will fail by buckling alone.

Euler's formula:

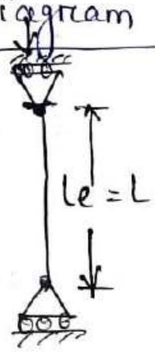
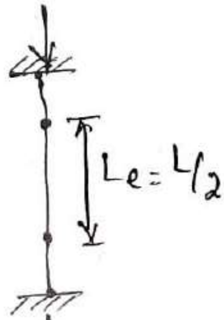
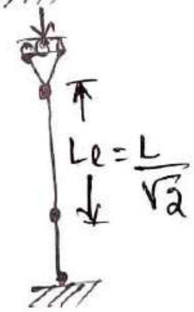
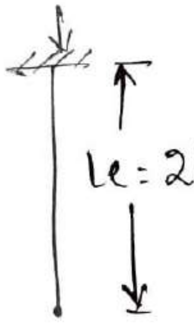
$$\boxed{P = \frac{\pi^2 EI}{l_e^2}}$$

where; l_e = Effective length

E = Young's modulus

I = Moment of Inertia of section a
of least resistance.

Effective length and critical loads for various boundary conditions :—

End conditions	Diagram	Effective length	Critica
Both ends hinged		L	$\frac{\pi^2 EI}{L^2}$
Both ends fixed		$\frac{L}{2}$	$\frac{4\pi^2 E}{L^2}$
One end fixed and other end hinged		$\frac{L}{\sqrt{2}}$	2
One end fixed and other end free		$2L$	π

As fc and E are constant for a particul
Euler's formula is valid for a particu
Slenderness ratio;
Ex- for mild steel whose $f_c = 3300 \text{ kg/cm}^2$
Euler's formula is not valid for slendernes
80.

(ii) Euler's formula is valid only up to prop
i.e. in linear elastic zone.

—x—

NOTE
- The relation between slenderness ratio
Critical Stress is hyperbolic.
- According to Euler's formula the critica
depend upon strength of material.
- The only material property involved is
modulus 'E' which physically represents
characteristics of the material.

Short Column:-

A short column of external diameter 'D' an
diameter 'd' is subjected to a load 'W'
eccentricity 'e' causing zero stress at an
the value of 'e' must be;

→ for a hollow circular cross-section;

$$\boxed{e \leq \frac{D^2+d^2}{8D}}$$

→ for a solid circular section;

$$\boxed{e \leq \frac{D}{8}}$$

Different formulaes for calculating limit of loads on columns:—

Rankine's formula;

$$\boxed{P_R = \frac{f_c A}{1+\alpha \lambda^2}}$$

P_R = Load

α = Rankine's constant.

f_c = yield stress.

λ = Slenderness ratio.

Straight line formula;

$$\boxed{P = A\left[f - n(\lambda)\right]}$$

P = Safe load on the column

A = Cross sectional area of column

f = Allowable stress in column materi

n = Constant which depends on the ma

λ = slenderness ratio.

Parabolic formula

$$\boxed{P = A\left[f - B\lambda^2\right]}$$

P = Safe load on the column

A = Cross sectional area of column

f = Allowable stresses of the column

λ = Slenderness ratio.

$$M = Pe \sec\left(l_e \sqrt{\frac{P}{EI}}\right)$$

σ_{max} = Critical stress on column

P = Axial load on the column

e = Eccentricity of the column load

l_n = Effective length of column.

EI = flexural rigidity.

Rankine's method;

$$\boxed{P = \frac{fA}{\left(1 + \frac{e y_c}{K^2_{min}}\right)\left[1 + \alpha \lambda^2\right]}}$$

P = Rankine's load

f = Allowable crushing strength of

e = Eccentricity of loading.

y_c = Distance of compression fibre

λ = slenderness ratio.

K min = Least radius of gyration w
minor axis.

Secant formula;

for standard pinned column;

$$\sigma_{max} = \frac{P}{A}\left(1 - \frac{e \cdot y_c}{K^2_{min}} \sec\left(\right.\right.$$

σ_{max} = Maximum stress is located at the
Compression fiber of the middle
of the column.

dimensions:-

(i) flanges = 150mm × 10mm.

(ii) Web = 280 mm × 10mm.

(iii) Overall depth = 300mm.

The column is hinged at one end & fixed having length of 5 m.

Calculate Safe load by using both Euler formula.

Take $E = 2 \times 10^5 \; N/mm^2$.

$\sigma_c = 320 \; N/mm^2$.

factor of Safety = 3 (FOS).

$\alpha = \frac{1}{7500}$ (Rankines Constant

Given; I - section.

(i) flanges = 150 mm × 10mm.

(ii) web = 280 mm × 10 mm.

(iii) overall depth = 300mm.

→ Column is hinged at one end & fixed at ot

$L = 5m = 5000mm$.

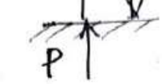
(i) Psafe = ?

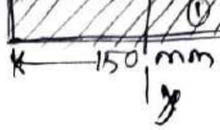
* Euler's formula.
* Rankine's formula.

$E = 2 \times 10^5 \; N/mm^2$.

$\sigma_c = 320 \; N/mm^2$.

$\alpha = \frac{1}{7500}$ (Rankine's constant.)

the Column will be; 



$$\boxed{L_e = \frac{L}{\sqrt{2}}}$$

$$\therefore \; L_e = \frac{5000}{\sqrt{2}}$$

$$L_e = 3535.53 \text{ mm.}$$

$\therefore$ Cross-Sectional area of given column:-

$$A = 2 \times (150 \times 10) + (280 \times 10)$$

$$= 5800 \text{ mm}^2.$$

Now $\because$ Euler's Buckling load is given by;

$$\boxed{P_E = \frac{\pi^2 E \, I_{min}}{(L_e)^2}} \longrightarrow (i)$$

$\therefore$ MI for given I-Section:-

(i) $$I_{xx} = \frac{BD^3}{12} - \left[2 \times \frac{bd^3}{12}\right]$$

$$\Rightarrow \frac{150(300)^3}{12} - \left[\frac{2 \times 70(280)^3}{12}\right]$$

$$\therefore I_{xx} = 81.4 \times 10^6 \text{ mm}^4.$$

(ii) $$I_{yy} = \left(\frac{db^3}{12}\right)_1 + \left(\frac{db^3}{12}\right)_2 + \left(\frac{db^3}{12}\right)_3$$

$$= \frac{10 \times 150^3}{12} + \frac{280 \times 10^3}{12} + \frac{10 \times 150^3}{12}$$

$$\therefore I_{yy} = 5.648 \times 10^6$$

$$\boxed{\because I_{xx} > I_{yy}}$$

$$\therefore I_{min} = I_{yy} = 5.648 \times 10^6 \text{ mm}^4.$$

$\therefore$ Safe Load $= \frac{P_E}{fos} \Rightarrow$ ~~891.90 KN~~. $\frac{89}{}$

$$\boxed{P_{safe} = 297.3\ KN}$$

By using Rankine's formula:-

Rankine's crippling load is given

$$\boxed{P_R = \frac{\sigma_c A}{1 + a\left(\frac{Le}{K}\right)^2}} \text{ ——— (2)}$$

$\therefore$ Radius of gyration $(K) = \sqrt{\frac{I_{min}}{A}}$

$\therefore K^2 = \frac{I_{min}}{A}$

$\Rightarrow \frac{5.648 \times 10^6}{5800}$

$\therefore K^2 = 973.79\ mm^2$.

$\therefore 1 + a\left(\frac{Le^2}{K^2}\right) = 1 + \frac{1}{7500}\left(\frac{3535.53^2}{973.79}\right.$

$\therefore 1 + a\left(\frac{(Le)^2}{K^2}\right) = 2.71$.

Put all values in eq no. 2.

$P_R = \frac{320 \times 5800}{2.71}$

$\therefore P_R = 687.87 \times 10^3 N$ or 687.87 K

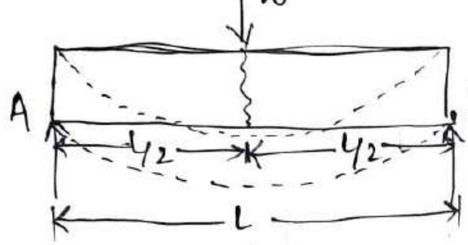
$\left[\therefore \text{Safe load} = \frac{P_R}{Fos} \Rightarrow \frac{687.87}{3} = 228.29\right.$

2. Usually long, straight and prismatic.

3. In most cases, loads are perpendicular to the beam.

4. This transverse loading causes only bendin in the beam.

5. When loads are not transverse, they also loads.



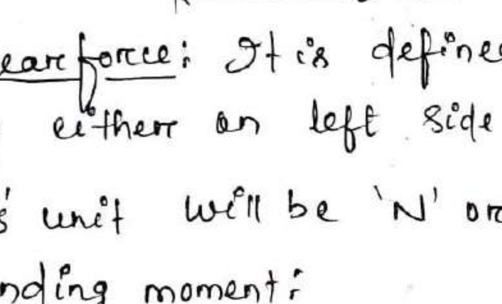
1. Shear force: It is defined as the algebraic acting either on left side or right side o

→ It's unit will be 'N' or 'KN'.

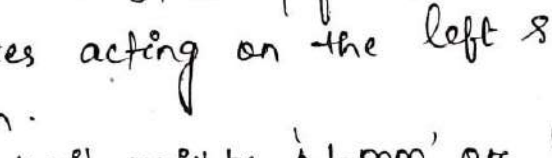
2. Bending moment:
It is defined as the algebraic su of forces acting on the left side or right section.

→ It's unit will be 'N-mm' or 'KN-mm'.

Imp. Sign convention for shear force:—



Sign convention for Bending moment!—



Hogging) BM.

is drawn below ...

3) for simply supported Beam, BM is zero ...
Supports.

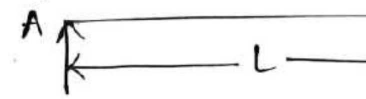
4) for cantilever beam, BM will be zero at ...

5) Calculate SF and BM at all critical poin...

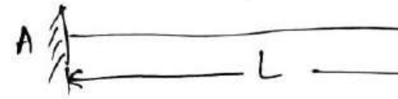
6) If no load is present between two points ...
SF will be constant.

Types of beams:-

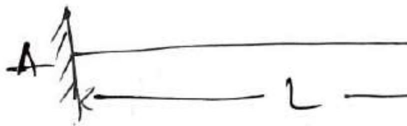
(i) Simply supported beam:-



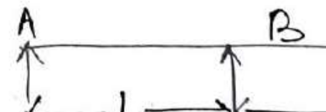
(ii) cantilever beam:-

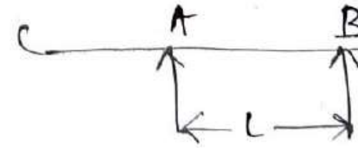


(iii) fixed beam:-

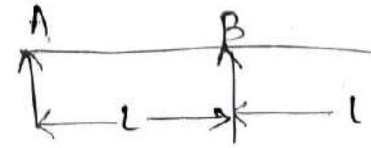


iv) Over hanging beam:-

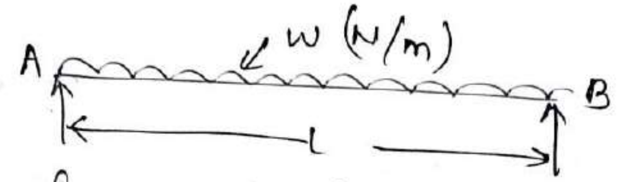




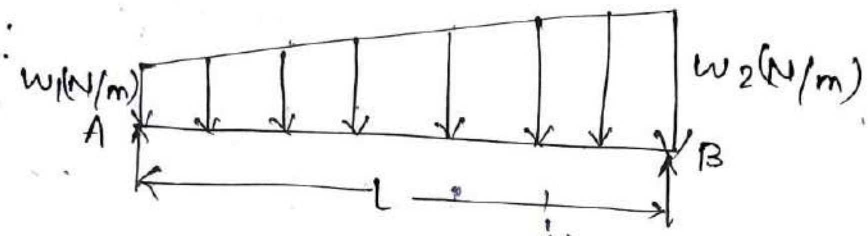
) Continous beam:-

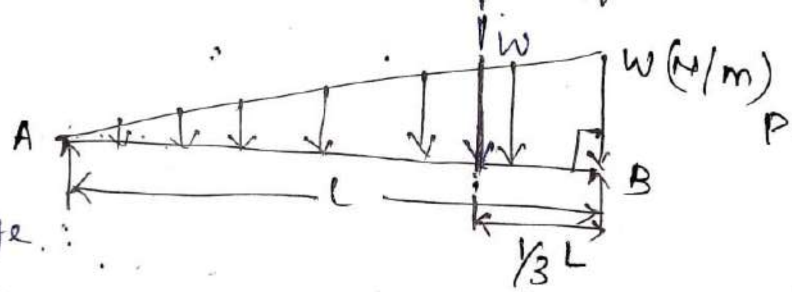


(ii) Uniformly distributed load :- (UdL)



(iii) Uniformly varying load :- (UVL)





point lo

Last page

Point

Problems

Q For an overhanging beam shown in fig. Support reactions at A & B. Draw SFD & B... find point of contraflexure.

Given data;

(i) R_A & R_B = ?

(ii) SFD & BMD

(iii) Point of contraflexure = ?

In BMD, the point at which its sign from positive to ... or negative to positive is ... Point of contraflexure ...

Sol^n 1) Calculation of support reaction :-

(i) $\Sigma f_y = 0$ ($\uparrow$ +ve & $\downarrow$ -ve)

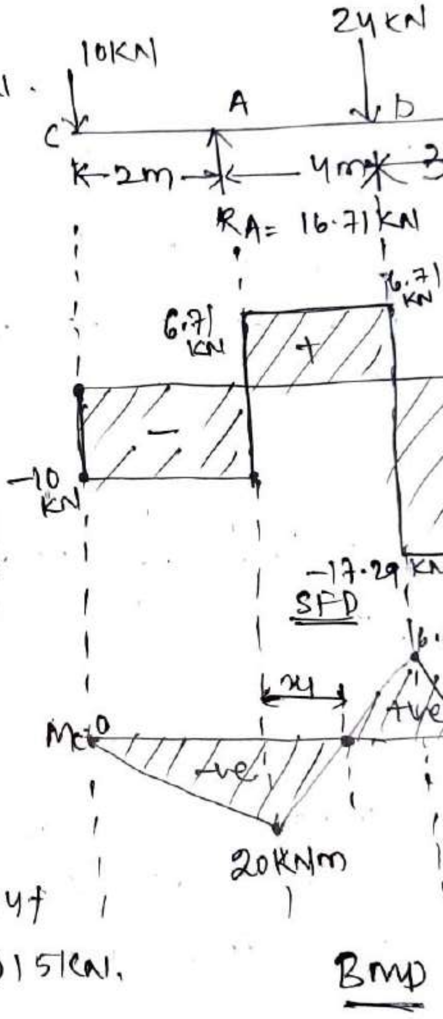
$(-R_B \times 7) + (15 \times 10) + (24 \times 4) - (10 \times 2) = 0$

$R_B = 32.29\ KN$.

Put $R_B = 32.29\ KN$ in eq(i).

$\therefore\ R_A + 32.29 = 49$

$\Rightarrow R_A = 49 - 32.29 = 16.71\ KN$.



2) SF calculations: −

+ve | −ve

(i) SF at point C = −10 KN.

(ii) SF at point A = $-10 + 16.71$
$= 6.71\ KN$.

(iii) SF at point D = $-10 + 16.71 - 24$
$= -17.29\ KN$.

(iv) SF at point B = $-10 + 16.71 - 24 + 39.29 \Rightarrow 15\ KN$.

v) SF at point E = 15 KN.

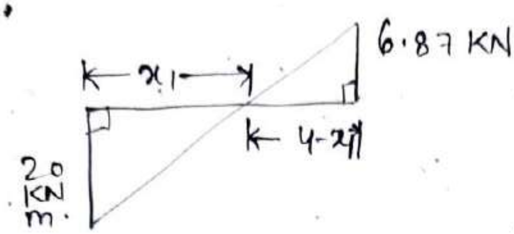
3) BM calculations:

(i) $M_C = M_E = 0$ [$\because$ it is overhanging beam]

+ve | −ve

(ii)

4) Location of point of contraflexure.



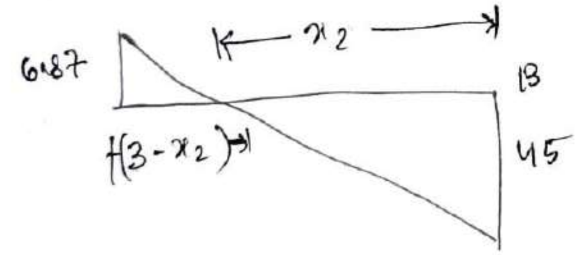
$$\frac{20}{x_1} = \frac{6.87}{(4-x_1)}$$

$\therefore\ 20(4-x_1) = 6.87\,x_1$

$\therefore\ 80-20x_1 = 6.87\,x_1 \Rightarrow 20x_1 + 6.87x_1 = 80$

$\Rightarrow x_1 = 80/26.87$

$\Rightarrow x_1 = 2.8$ m from point A.



$$\frac{6.87}{(3-x_2)} = \frac{45}{x_2}$$

$\Rightarrow 6.87\,x_2 = 45\,(3-x_2)$

$\Rightarrow 6.87\,x_2 = 135 - 45x_2$

$6.87\,x_2 + 45x_2 = 135$

$\Rightarrow 51.87\,x_2 = 135$

$\Rightarrow x_2 = \frac{135}{51.87} = 2.6$ from p

$\therefore R_B + R_C - 200 - (180 \times 10) = 0$

$\therefore R_B + R_C = 2000\,N$ ——— (i)

(ii) $\Sigma M_B = 0$ (↻ +ve & ↺ -ve)

$\therefore -R_C \times 7 + (1800 \times 4) - (200 \times 1) = 0$

$\therefore R_C = \underline{1000\,N}$

Put $R_C = 1000\,N$ in eqn.

$R_B + 1000 = 2000$

$\Rightarrow R_B = \underline{1000\,N}$

2) <u>SF calculation</u> :

+ve -ve

↑↓ ↓↑

(i) SF at point A = -200N

(ii) SF at point B_L = -200 - 180
$\Rightarrow -380\,N$.

(iii) SF at point B_R = -200 - 180 + 1000
$\Rightarrow$ ~~-380N~~ 620N

(iv) SF at point C_L = -200 - 180 + 1000 - 1260 $\Rightarrow$ -640 N
(7 × 180)

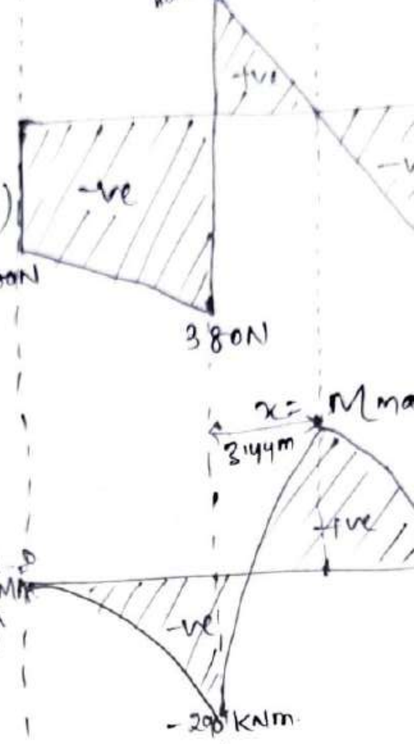
(v) SF at point C = -200 - 180 + 1000 - 1260 + 1000 $\Rightarrow$ + 360N

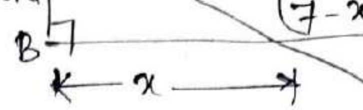
vi) SF at point D = 0

3) <u>Bending moment calculations</u> :-

(i) $M_A = M_D = 0$ [$\because$ overhanging beam]

(ii) $M_B = (-200 \times 1) - (180 \times 0.5) = -290\,KNm$.





$$\frac{620}{x} = \frac{640}{(7-x)}$$

$\Rightarrow 620(7-x) = 640x$

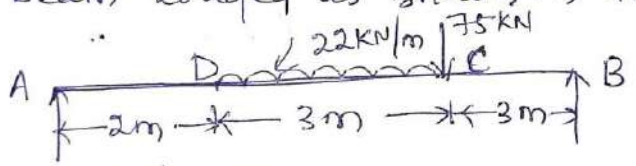
$\Rightarrow x = 3.44$ m from point B.

5) Calculation of maximum Bending moment :-

(i) Max BM $= (-200 \times 4.44) + (1000 \times 3.44)$

$\therefore$ Max BM $= 777.78$ KNm.

Q For the beam loaded as shown in fig :-



Draw SFD and BMD.

A Simply supported beam of span 6m carries 1.5 KN/m over entire span & a point load of from right Support. Draw SFD and BMD.

Given

1) Calculation of Support reactions :

(i) $\Sigma f_y = 0$ ($\uparrow$ +ve, $\downarrow$ -ve)

$\therefore R_A + R_B - 75 - (22 \times 3) = 0$

$\therefore R_A + R_B = 75 + 66 = 141$ —— (1)

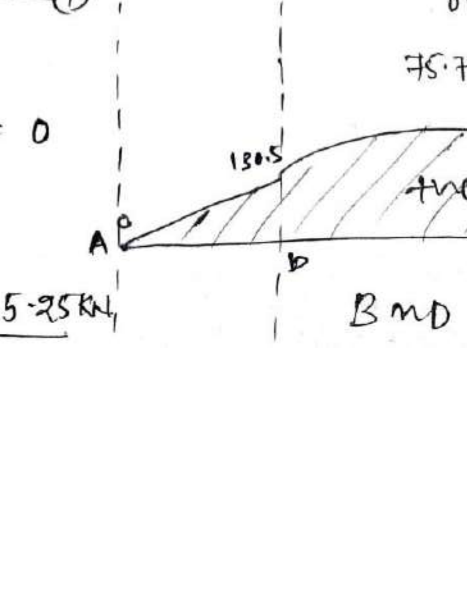
(ii) $\Sigma M_A = 0$ (↻ +ve & ↺ -ve)

$\therefore -R_B \times 8 + (75 \times 5) + 66 \times (2 + \frac{3}{2}) = 0$

$\therefore R_B = 75.75$ KN.

Put $R_B = 75.75$ KN in eq^n(i).

$\therefore R_A = 141 - 75.75 = 65.25$ KN



(iii) SF at point C = 65.25 - 66 - 75 → -75.75 K

(iv) SF at point B = -75.75 KN

3) BM Calculations

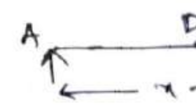
portion 'AD'

$M_x = R_A \cdot x$

@ $x = 0$, $M_A = 0$

@ $x = 2$, $M_D = 65.25 \times 2 = 130.5$ KNm.

portion 'DC'



$M_x = R_A \cdot x - 22 \times \frac{(x-2)^2}{2}$

@ $x = 2$, $M_D = 65.25 \times 2 - 0 = 130.5$ KNm.

@ $x = 5$, $M_C = 65.25 \times 5 - 22 \times \frac{(5-2)^2}{2} = 22$

portion 'CB'

$M_x = R_A \cdot x - 22 \times 3 \times (x - 3.5) - 75 \times (x - 5)$

@ $x = 5$, $M_C = 65.25 \times 5 - 22 \times 3 \times (5 - 3.5)$

$= 227.25$ KNm.

@ $x = 8$, $M_B = 65.25 \times 8 - 22 \times 3 (8 - 3.5) -$

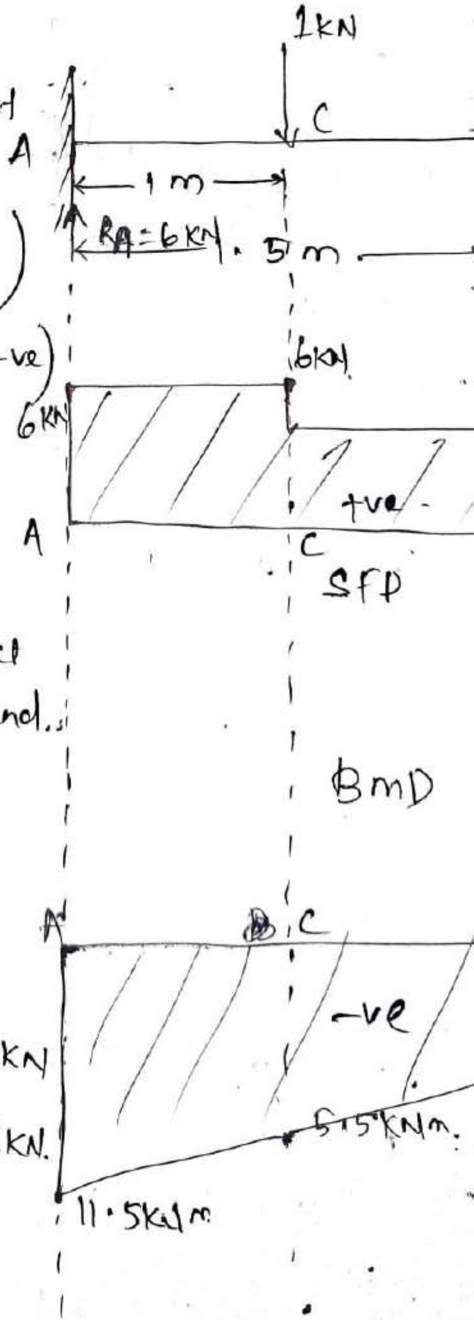
$= 0$

$M_D = 75.75 \times 6 - 75 \times 3 - 22 \times 3 \times \frac{3}{2} =$

$M_A = 75.75 \times 8 - 75 \times 5 - 22 \times 3 \times \left(\frac{3}{2} + 2\right).$

$\therefore = 606 - 375 - 231 = \underline{\underline{0}}$

Q Draw SFD and BMD for a cantilever given below..



Soln Calculation of Support reaction.

(i) $\Sigma f_y = 0$ (All the vertical force $= 0$)

$\therefore R_A - 1 - 2 - 3 = 0$ (↑ +ve & ↓ -ve)

$\therefore R_A = 6$ KN.

(ii) SF Calculation:

→ for cantilever beams start SF calculation from free end.

+ve ↑ ↓   -ve ↓ ↑

(i) SF at point B = 3KN.

(ii) SF at point D = 3+2 = 5KN

(iii) sf at point C = 3+2+1 = 6KN.

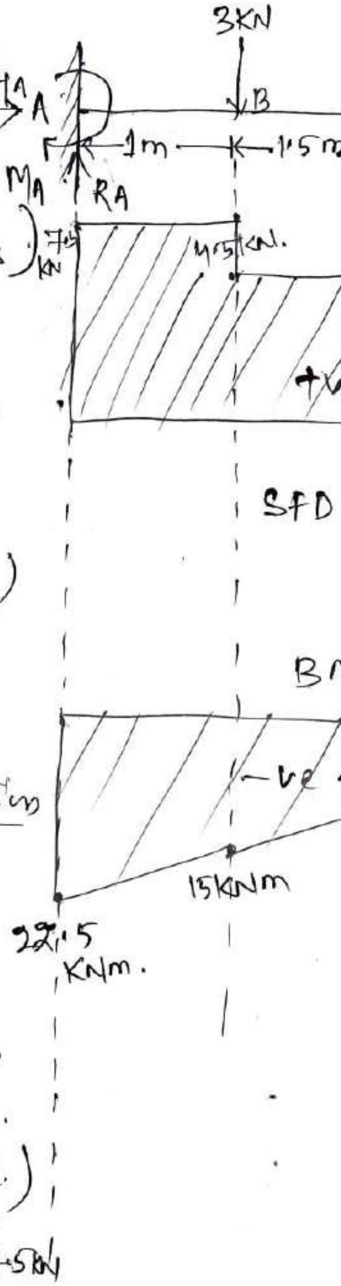
(iv) sf at point A = 6 KN.

(iii) $M_C = (-3 \times 1.5)(2 \times 0.5)$

$= -5.5$ KN m.

(iv) $M_A = (-3 \times 2.5) - (2 \times 1.5) - (1 \times 1)$

Q Draw SFD & BMD for the cantilever shown in figure.



1. Calculation of Support reaction:-

(i) $H_A = 0$ ($\because$ No horizontal forces)

(ii) $\Sigma F_y = (\uparrow +ve, \downarrow -ve)$

$R_A - 3 - 2.5 - (1 \times 2) = 0$

$\Rightarrow R_A = 7.5$ KN.

(iii) $\Sigma M_A = (2.5 \times 5) + (3 \times 1)$
$(1 \times 2 \times 3.5)$

$\therefore M_A = 22.5$ KN/m.

2. Shear force Calculation

+ve: $\uparrow \mid \downarrow$ -ve: $\downarrow \mid \uparrow$

i) SF at point E = 2.5 KN.
ii) SF at point D = 2.5 KN.
iii) SF at point C = 2.5 + (1 × 2) = 4.5 KN.
iv) SF at point B = 4.5 + 3 = 7.5 KN
v) SF at point A = 7.5 KN.

$(iii)\ M_C = -(2.5 \times 2.5) - \left(1 \times 2 \times \frac{2}{2}\right)$

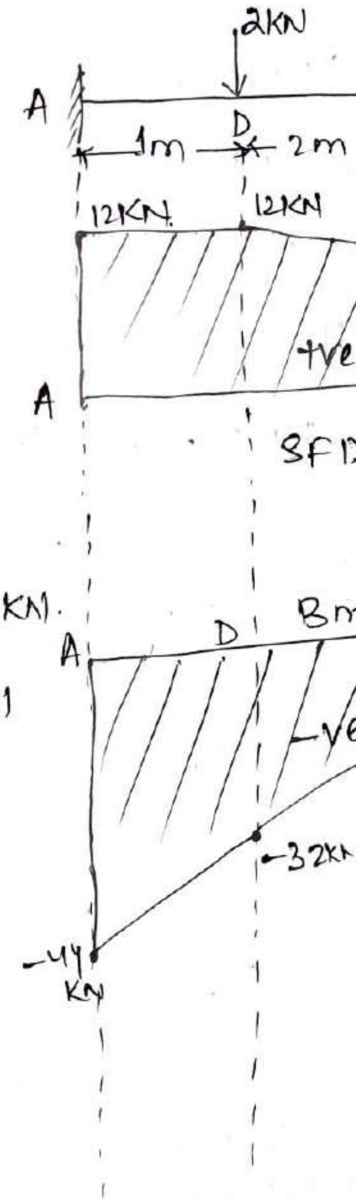
$= -8.25$ KN/m.

$(iv)\ M_B = -(2.5 \times 4) - ((1 \times 2) \times 2.5) =$

$(v)\ M_A = -(2.5 \times 5) - ((1 \times 2 \times 3.5)) - (3$

$= -22.5$ KN/m.

Q Draw SFD and BMD for a cantilever in figure.



$Sf_B = 6$ KN

$Sf_C = 6 + 4 = 10$ KN

$Sf_D = 6 + 4 + 2 + 12$ KN.

$Sf_A = 12$ KN.

$BM_B = 0$ ($\because$ free end)

$BM_C = -6 \times 2 = -12$ KN.

$BM_D = -6 \times 4 - 4 \times 2 = -32$ KN.

$BM_A = -6 \times 5 - 4 \times 3 - 2 \times 1$

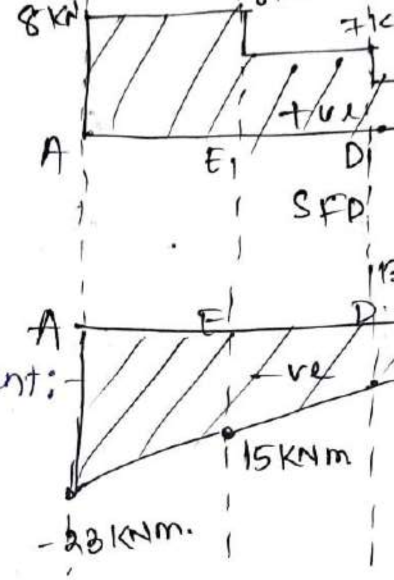
$= -44$ KN m.

$SF_C = 2 \times 2 = 4\,KN$

$SF_D = 4 + 3 = 7\,KN$

$SF_E = 7 + 1 = 8\,KN$

$SF_A = 8\,KN$.



Calculation of Bending moment:-

$BM_B = 0$

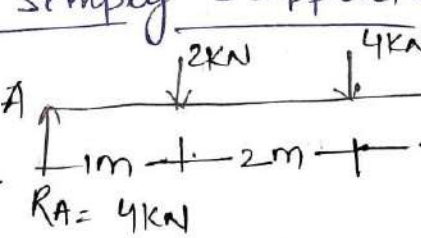
$BM_C = -4 \times 1 = -4\,KNm$.

$BM_D = -4 \times 2 = -8\,KNm$

$BM_E = -4 \times 3 - 3 \times 1 = -12 - 3 = -15\,KNm$.

$BM_A = -4 \times 4 - 3 \times 2 - 1 \times 1 = -23\,KNm$.

Q SFD and BMD for Simply Supported



) Sum of upward force = sum of downward force.

$R_A + R_B = 2 + 4 + 2$

$R_A + R_B = 8\,KN$ —— (1).

) $\Sigma M_A = 0$

$R_B \times 6 - 2 \times 5 - 4 \times 3 - 2 \times 1 = 0$.

$6R_B = 10 + 12 + 2$.

$\Rightarrow 6R_B = 24$

$\Rightarrow R_B = \frac{24}{6} = 4\,KN = R_B$

from (1) : $R_A = 4\,KN$.

$SF_E = 2 + 2 = 4KN$

$SF_A = 4KN$.

Bending moment calculation:-

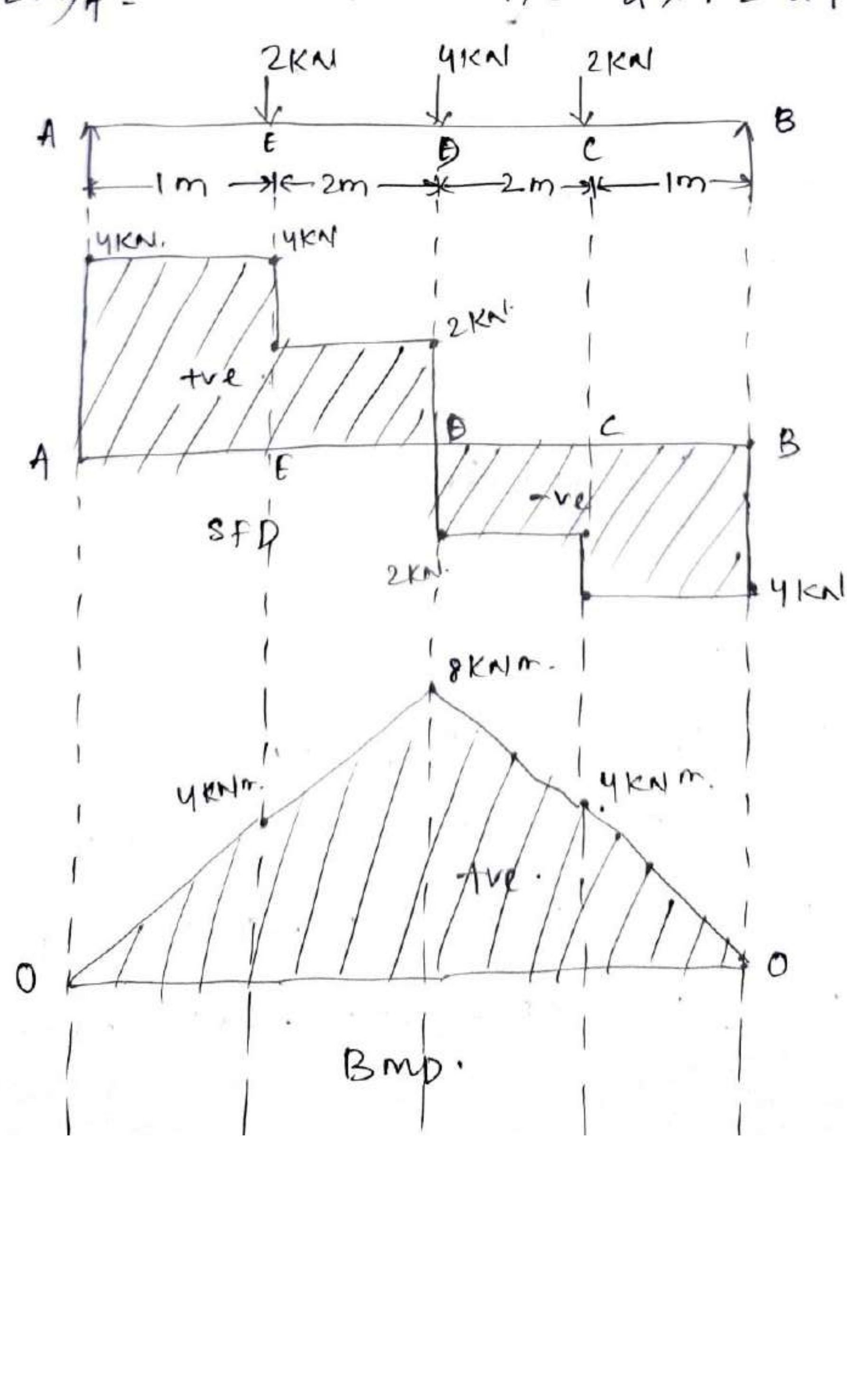
$(BM)_B = 0$

$(BM)_C = 4 \times 1 = 4KNm$.

$(BM)_D = 4 \times 3 - 2 \times 2 = 12 - 4 = 8 KNm$.

$(BM)_E = 4 \times 5 - 2 \times 4 - 4 \times 2 = 20 - 8 - 8$

$(BM)_A = 4 \times 6 - 2 \times 5 - 4 \times 3 - 2 \times 1 = 24 -$



... of downward force.

$R_A + R_B = 2 + 6 + 4 = 12\ kN$ — ①

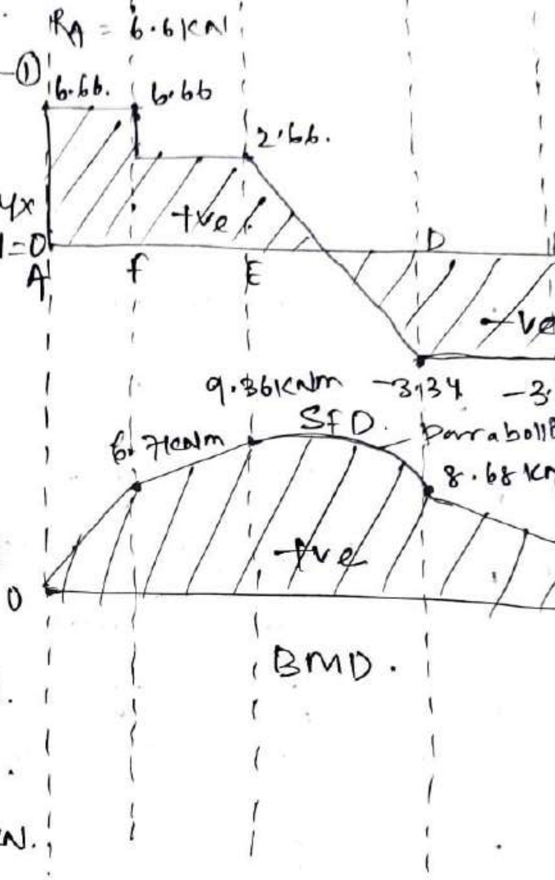
② $\Sigma m_A = 0$.

$R_B \times 6 - 2\times5 - 6\times3 - 4\times1 = 0$

$\Rightarrow R_B \times 6 = 32$

$\Rightarrow R_B = 32/6 = 5.34\ kN.$

$R_A = 12 - 5.34 = 6.6\ kN.$



③ Shear force Calculation:

$SF_B = -5.34\ kN.$

$SF_C = -5.34 + 2 = -3.34\ kN.$

$SF_D = -5.34 + 2 = -3.34\ kN.$

$SF_E = -5.34 + 2 + 6 = 2.66\ kN.$

$SF_F = -5.34 + 2 + 6 + 4 = 6.66\ kN.$

$SF_A = 6.66\ kN.$

④ Bending moment Calculation:

$(BM)_B = 0$

$(BM)_C = 5.34\times1 = 5.34\ kNm.$

$(BM)_D = 5.34\times2 - 2\times1 = 8.68\ kNm$

$(BM)_E = 5.34\times4 - 2\times3 - 6\times1 = 9.36\ kNm$

$(BM)_F = 5.34\times5 - 2\times4 - 6\times2 = 6.7$

$(BM)_A = 5.34\times6 - 2\times5 - 6\times3 - 4\times1 = 0\ kN$

Taking moment about point 'A'. $\Sigma M = 0$.

$R_B \times 6 - 50 \times 4 - 20 \times 2 \times 2$.

$\Rightarrow R_B \times 6 = 50 \times 4 + 20 \times 2 \times \frac{2}{2}$

$= 200 + 40$

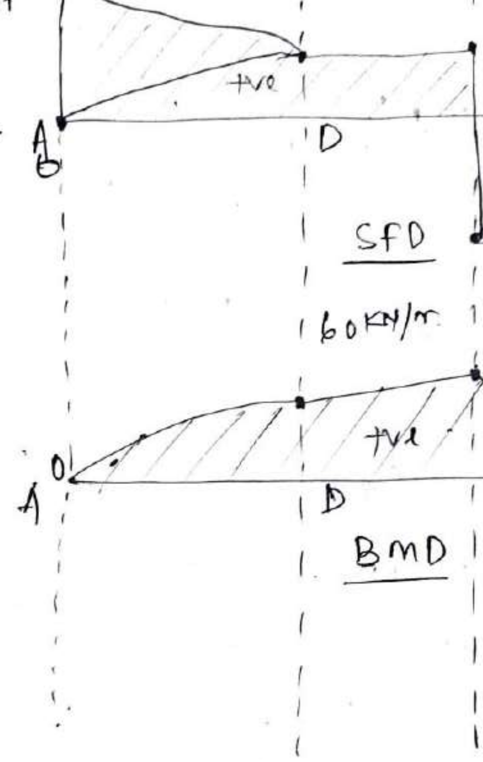
$= 240$

$\Rightarrow R_B = \frac{240}{6} = \cancel{46.66}\ 40$.

$R_A + 40 = 90\,KN$.

$\Rightarrow R_A = 50\,KN$.

$R_B = 40\,KN$



Shearforce Calculation:

Sf at point B $= -40\,KN$.

$(Sf)_C = -40 + 50 = 10\,KN$.

$(Sf)_D = -40 + 50 = 10\,KN$.

$(Sf)_A = -40 + 50 + 40 - 50\,KN = 0\,KN$

Bending moment Calculation:

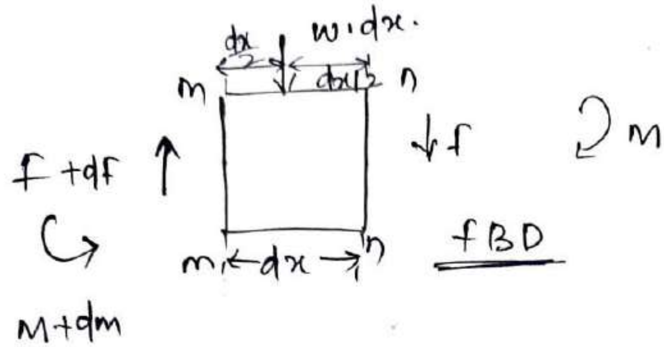
$(BM)_B = 0$

$(BM)_C = 40 \times 2 = 80\,KN/m$.

$(BM)_D = 40 \times 4 - 50 \times 2 = 160 - 100 = 60\,K$

$(BM)_A = 40 \times 6 - 50 \times 4 - 40 \times 1 = 240 - 200$

$= 0\,KN/m$.



FBD

(i) $\Sigma f_y = 0$ ($\uparrow$ +ve & $\downarrow$ -ve)

$\therefore \cancel{-f} + \cancel{f} + df - w\,dx = 0$

$df = w\,dx.$

$\therefore \boxed{w = \frac{df}{dx}}$ —— ① (~~Relation bet^n~~ loading

eq^n (i) gives the relation between the int... loading and shear force.

(ii) $\Sigma M_{|mm} = 0$ (↻ +ve & ↺ -ve)

$\therefore \cancel{M} + dm - \cancel{M} - f\,dx - w\,dx \cdot \frac{dx}{2} = 0$

$\therefore dm - f\,dx - \cancel{\frac{w\,dx^2}{2}}^{\,0} = 0$

$dm = f\,dx$

$\Rightarrow \boxed{f = \frac{dm}{dx}}$ —— Eqn(2)

Eq^n (ii) gives the relation between SF & ...

f = Amount of shear force acting on the ...

dm = Amount of BM.

dx = distance.

limit considered.

→ Now the bending may be pure or non-unifo
Under both conditions deflections are produ

→ The deflections also occur due to tempera
and lack-of-fit of members.

→ The deflections of structures are importan
that the designed structure is not excessivel
The large deformations in the structures ca
or cracking of non-structural elements.

→ For statically indeterminate structures, d
deflection conditions are used in addition
conditions for determination of unknown
reaction.

* statically determinate structure is a s
static and all unknown reactive forc
determined by the equilibrium equations
$\Sigma F_y = 0$ and $\Sigma M = 0$).

* Statically indeterminate :- When the sta
equations; force & moment equilibrium c
are insufficient for determining the in
and reactions on a structure.

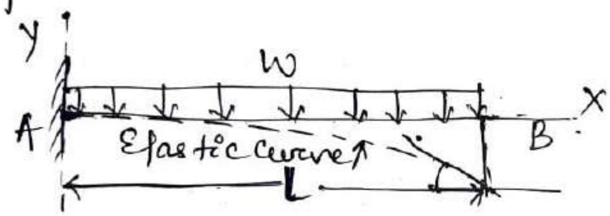
The deflection of beam depends on four g

1. Stiffness of the material that the b
2. Dimension of the beam.
3. Applied loads.
4. Support conditions.

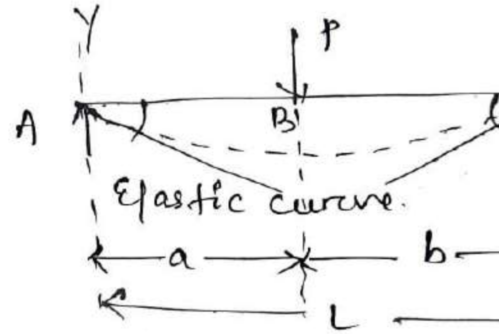
→ The curve into which the axis of the beam is tra[...] under the given loading is called the elastic c[...]

→ the nature of the elastic curve depends on the s[...] conditions of the beam and the nature and type [...] loadings.

→ The slope at a given point may be clockwise [...] anticlockwise measured from the original ax[...] of the beam.



(a) Cantilever beam.



(b) Simply suppo[...] be[...]

figure shows the elastic curves for cantilev[...] and simply supported beams. Sagging or [...] positive bending moment produces an elastic [...] with curvature of concave upward where[...] hogging or negative bending moment [...] rise to an elastic curve with curvature [...] concave downward.

the deflection.

Slope:-

The angular displacement or rotation of the
drawn at a point on the elastic curve of a
respect to the ~~original~~ longitudinal axis of t
beam without loading is known as the sl
given point.

Importance of slope and deflection:-

Accurate values for these beam deflections
in many practical cases. The deflection of a
must be limited in order to:

(a) provide integrity and stability of stru
machine.

(b) Minimize or prevent brittle-finish mate
cracking.

The computation of deflections at specific
structures is also required for analy
statically indeterminate structures.

General procedure for computing deflection

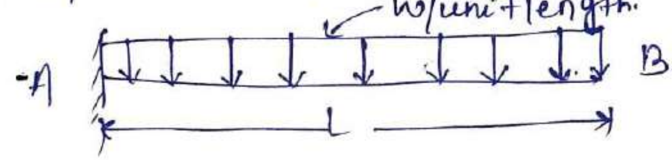
1. Select the interval or intervals of the bean
used and place a set of coordinate axis
with the origin at one end of an interva
indicate the range of values of x in ea

$$EI\left(\frac{d^2y}{dx^2}\right).$$

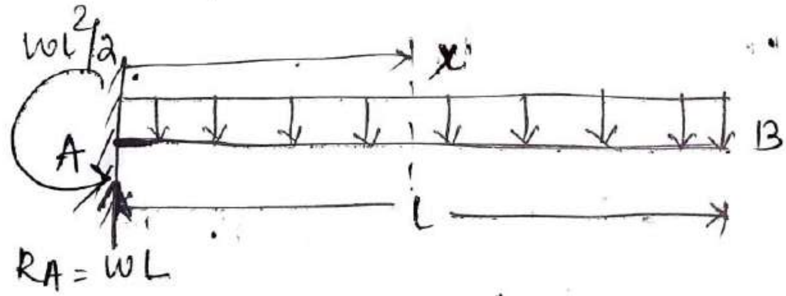
4. Solve the differential equation from step ... C... evaluate all constants of integration. ... slope (dy/dx) and deflection (y) at th... points.

Problem-1

A cantilever beam of length L carries a... distributed load of 'w' per unit length over... entire length. Determine the slope and d... at the free end of the beam.



Soln



Determine the support reaction:

Sum of the vertical forces,

$$\Sigma V = 0, \; R_A = wL$$

Taking moment about any section between the entire length of the cantilever,

We have $\quad M(x) = -\dfrac{wL^2}{2} - \dfrac{wx^2}{2} + w$...

$$EI\theta = EI\frac{dy}{dx} = \frac{-wL^2x}{2} - \frac{wx^3}{6} + \frac{wLx}{2}$$

Integrating again with respect to x, we get

$$EIy = \frac{-wL^2x^2}{4} - \frac{wx^4}{24} + \frac{wLx^3}{6} + C_1x$$

The constants integration C_1 and C_2 may be det from the boundary conditions.

$$x = 0, \ \theta = 0 \text{ and } x = 0, \ y = 0.$$

~~The constants integration C_1 and C_2 may be d from the boundary conditions.~~

$\cancel{x = 0, \theta = 0, \text{ and}}$

Substituting $x = 0$, $\theta = 0$, in eq^n(i) we ge

Substituting $x = 0$, $y = 0$, in eq^n(ii), we ge

Substituting the values of $C_1 = 0$ & $C_2 = 0$ in

we get;

General eq^n for slope;

$$EI\theta = EI\frac{dy}{dx} = \frac{-wL^2x}{2} - \frac{wx^3}{6}$$

General eq^n for deflection;

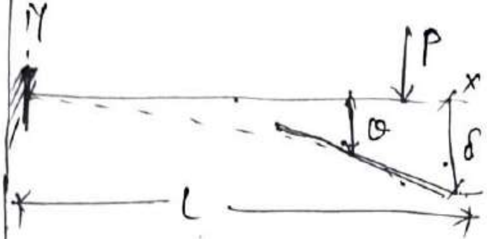
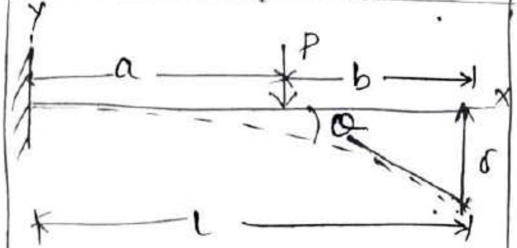
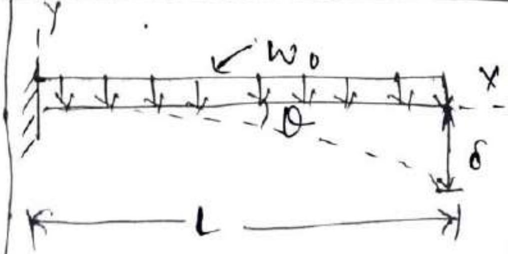
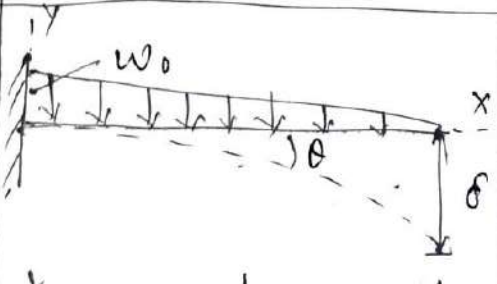
$$EIy = \frac{-wL^2x^2}{4} - \frac{wx^4}{24} + \frac{wLx^3}{6}$$

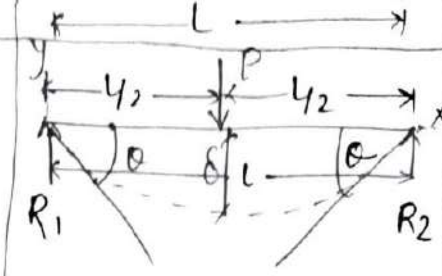
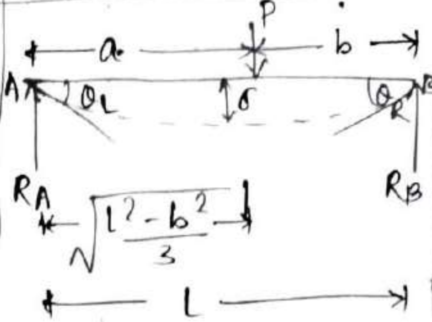
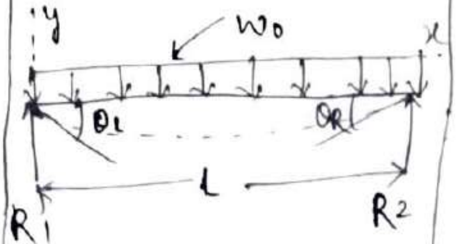
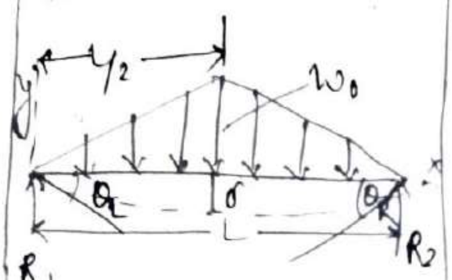
Slope at free end $(x = L)$

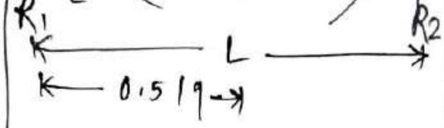
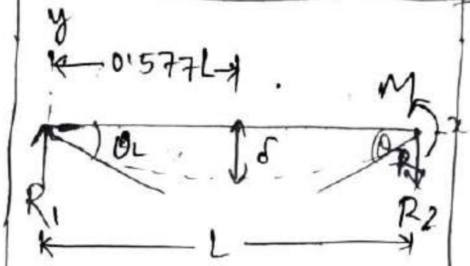
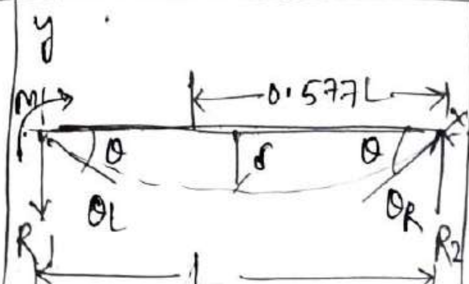
$$EI\theta_B = -\frac{wL^3}{2} - \frac{wx^3}{6} + \frac{wL^3}{2}.$$

$$\boxed{\theta_B = -\frac{wL^3}{}}$$

$$\boxed{y_B = -\frac{WL^3}{8EI}}$$

Slno	Types of load	Maximum moment + sagging	Slope at en
1.		$M = -PL$	$\theta = \frac{PL^2}{2EI}$
2.		$M = -Pa$	$\theta = \frac{Pa^2}{2EI}$
3.		$M = \frac{w_0 L^2}{2} = -\frac{WL}{2}$ where $W = w_0 a$	$\theta = \frac{w_0 L^3}{6EI} = \frac{WL^2}{6EI}$
4.		$M = -M$	$\theta = \frac{ML}{EI}$

6.		$M = \frac{PL}{4}$	$\theta_L = \theta_R =$
7.		$M = \frac{Pab}{L}$ at $x = a$	$\theta_L = \frac{Pb(L}{6E}$ $\theta_R = \frac{Pa(L^2}{6E}$
8.		$M = \frac{w_0 L^2}{8}$ $= \frac{WL}{8}$ Here, $W = w_0 L$	$\theta_L = \theta_R =$ $\frac{w_0 L^3}{24EI}$
9.		$M = \frac{w_0 L^2}{12}$ $= \frac{WL}{6}$	$\theta_L = \theta_R =$ $\frac{5 w_0 L^3}{192EI}$

		$= \frac{2WL}{9\sqrt{3}}$ Hence, $W = \frac{W_0 L}{2}$	CL 2CL
11		$M = M$	$\theta_L = \frac{M}{6E}$ $\theta_R = \frac{M}{3E}$
12		$M = M$	$\theta_L = \frac{ML}{3EI}$ $\theta_R = \frac{ML}{6EI}$

by a single function of $M(x)$. However
always the case. When the loading of the
such that two or more functions are n
represent the bending moment over the
of the beam. ~~As~~

In such cases, additional constants of
and as many numbers of equations
necessary to express continuity condition
points of load change-over in addition t
boundary conditions. Thus the process
lengthy and cumbersome. To overcome th
British engineer W.H. Macaulay propo
innovative approach of solving such p
using singularity function to express
moment over the entire length.

$$\boxed{\theta_C = -\frac{WL^2}{32EI}}$$

$$\boxed{y_C = -\frac{3WL^3}{256EI}}$$

* The structure which can be analysed by equation of equilibrium alone is said a determinate structure.

Analysis means → we have to find the no. reactions and values of unknown internal fo the help of equilibrium equation.

* In Determinate structure stress / force be developed due to temperature effects, fit, and support settlement.

* The SF and BM values doesnot depends cross-sectional area.

<u>Statically indeterminate structure:-</u>

The structure which can't be analysed by of equilibrium equation alone is known indeterminate structure.

So the analysis of this type of struct need some additional eqn are required are known as <u>Compatibility equation</u>.

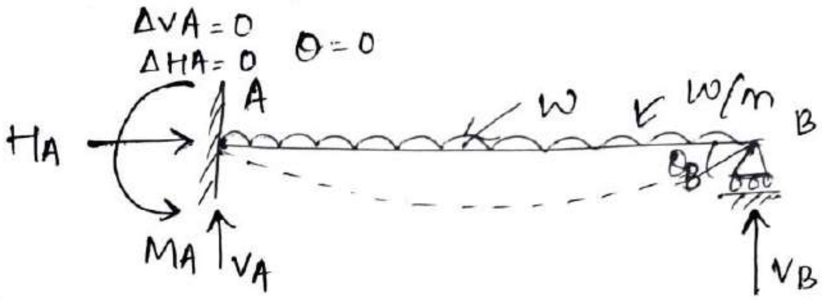
* Here in indeterminate structure ther additional stress / force will be develope temperature effects, lack of fit an support settlement.

- Deformation = Δ, θ.

Δ = Represent displacement of var

θ = Represents rotation of variou

Ex



Pro

ΔV

- Let us Consider 1st the support 'B'.
- In Support 'B' there will be a ro
  i.e, θ_B. due to roller Support.
  (Because as we know that. in roller
  is allowed.).

But at the joint 'B' movement in hori
is allowed but the movement in ver
is not allowed that's why there w
in vertical direction.

So, Here at joint 'B', Vertical displace

i.e, $\boxed{\Delta V_B = 0}$ ← this is the
Compacti

Here the displacement will be -

(1) Downward displacement.

(2) Upward displacement.

Here $\Delta V_B = 0$ Means
- upward displacement = Downward displace

- In case of joint 'A'
  there will be a vertical reaction, So ΔV_A
  there will be a Horizontal reaction, so ΔH_A
  there will be a Moment, So $\theta = 0$

$$\boxed{\begin{aligned} \Delta V_A &= 0 \\ \Delta H_A &= 0 \\ \theta &= 0 \end{aligned}}$$

These are three Compatibility equa at joint 'A'.

So $\Delta V_B = 0$ $\quad \Delta H_A = 0$
$\Delta V_A = 0$ $\quad \theta = 0$

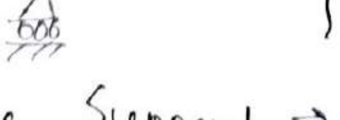
we use these Compatibi equation for finding the unknowns where equilib Condition are not suffici find the unknowns.

Note.

No. of Compatibility Condition depends upo no. of reaction at the Support.

<u>No. of compatibility e</u>

① for Roller Support →



1 ($\Delta V = 0$)

② for Hinge Support →

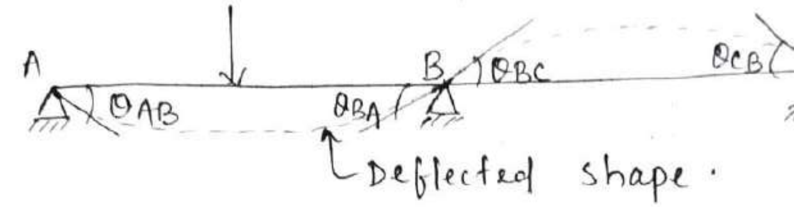
2 ($\Delta V = 0, \Delta$

③ for fixed Support →

3 ($\Delta V = 0, \Delta H$
$\theta = 0$

No. of Compatibility → Depends upon
at the Support

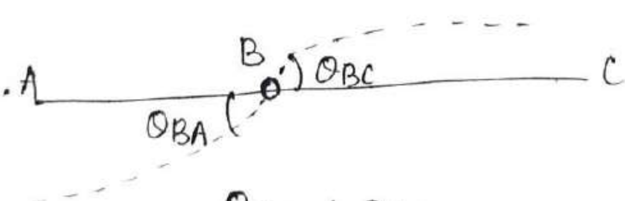
Compatibility condition for Rigid jo



$\theta_{BA} = \theta_{BC}$ → (As a rigid joint)

It is also a compatibility c

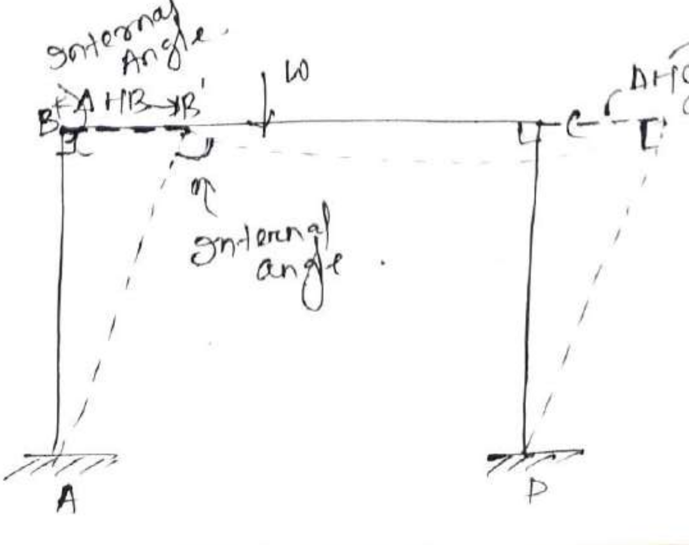
If the joint 'B' is a pin joint then,



$\theta_{BA} \neq \theta_{BC}$

this is a not comp

for frames



For Rigid joint the internal angle ...

$\boxed{\Delta_{HB} = \Delta_{HC}.}$

this is the compatibility condition

(for pin joint the internal angle will be not same

If the joints will pin-joint then.

$\boxed{\Delta_{HB} \neq \Delta_{HC}.}$

This is not the compatibility condition.

Analysis of propped and fixed beams:-

When the free end of a cantilever is supported a hinge or roller support then the beam is propped beam.

There are several methods to find the value of fixing bending moments. the following are usual method:-

1. Moment area method
2. Macaulay's method
3. moment distribution method
4. three moment method
5. method of flexibility co-efficients.

Equations of static equilibrium are:

$$\Sigma f_x = 0.$$

$$\Sigma f_y = 0$$

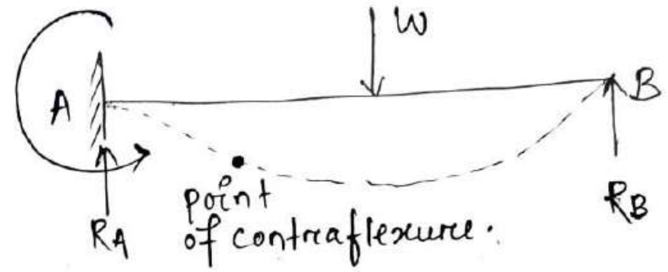
$$\Sigma M = 0.$$

NOTE

$\Sigma f_x = 0$ shall be considered only if in
horizontal loads exist.

Important cases of static indeterminacy:

Propped Cantilever:—



unknowns ; R_A , R_B and M_A.

$\Rightarrow$ No. of unknowns (R) = 3

Useful equilibrium equations: $\Sigma f_x = 0$

$\Rightarrow$ No. of useful equilibrium equation (E

$\therefore$ static indeterminacy $= R - E = 1$.

$\therefore$ for complete analysis, one additi
bility equation is considered by e
upward deflection due to prop ar
deflection due to external load at

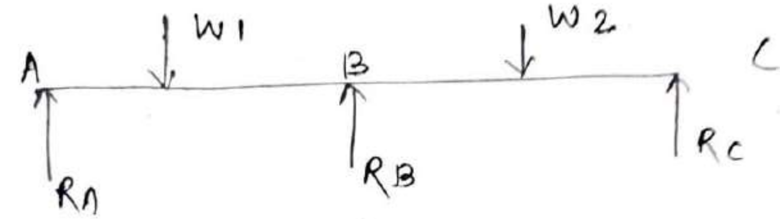
R_A point of contraflexure

No. of unknowns (R) = 4

No. of useful equations (l) = 2

∴ static indeterminacy or redundancy =

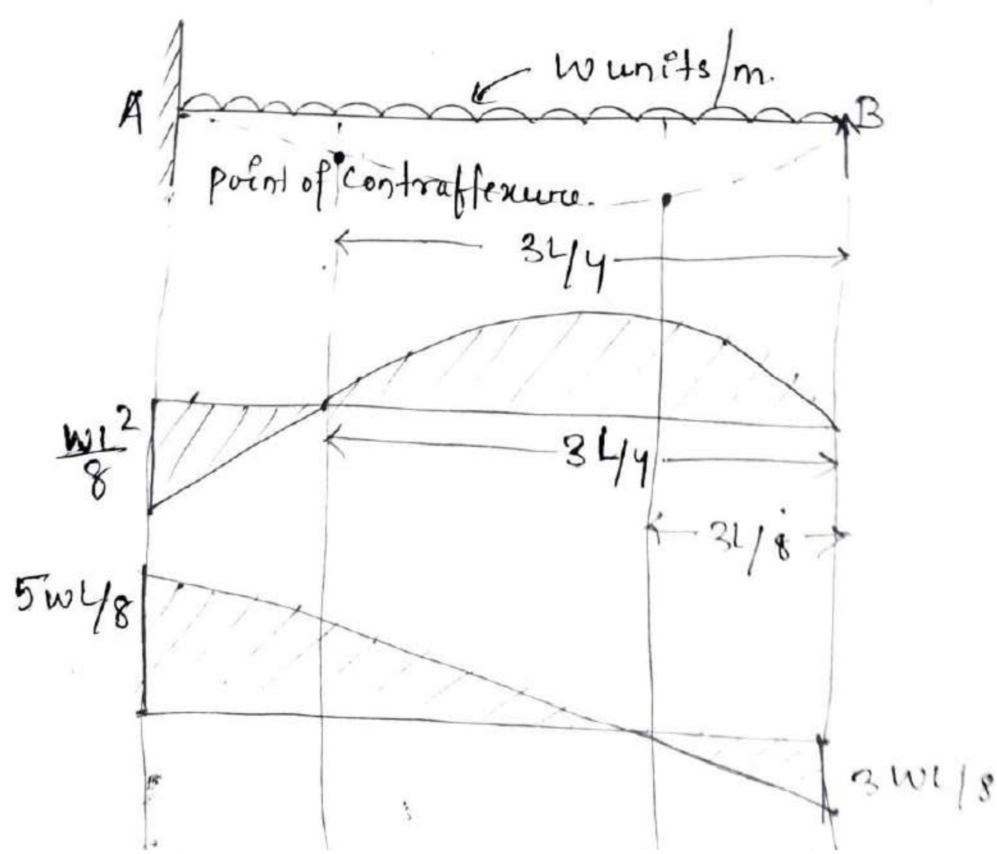
- <u>Continuous Beams extra (Support at B):-</u>



Static indeterminacy = 1.

<u>Analysis:-</u>

- <u>Propped Cantilever with u.d.l. through</u>

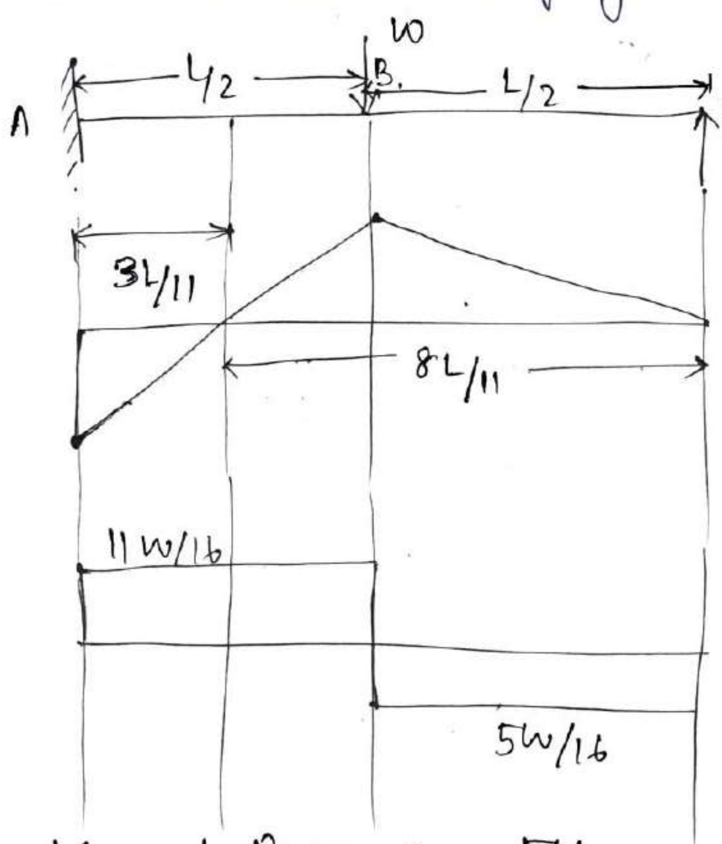


$R_B = \frac{\quad}{8}$

Maximum positive BM $= \frac{9wL^2}{128}$

Support moment $= \frac{wL^2}{8}$ (hogging) .

Propped Cantilever Carrying central



Reaction at Prop, $R_B = \frac{5W}{16}$

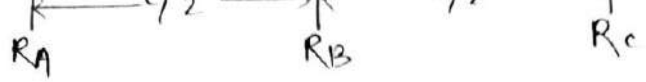
moment at fixed end, $M_A = -\frac{3WL}{16}$ (ho

Maximum bending moment,

$BM_{max} = \frac{5WL}{32}$ (sa

Point of Contraflexure :-

At $\frac{3L}{11}$ from fixed Support .



Reaction at prop. $R_B = \frac{5wL}{8}$

Bending moment at B, $M_B = \frac{wL^2}{32}$ (hogging)

Analysis of fixed beams:—

Support moments (By moment area method)

(i) Area of free and fixed B.M.D's are num
equal.

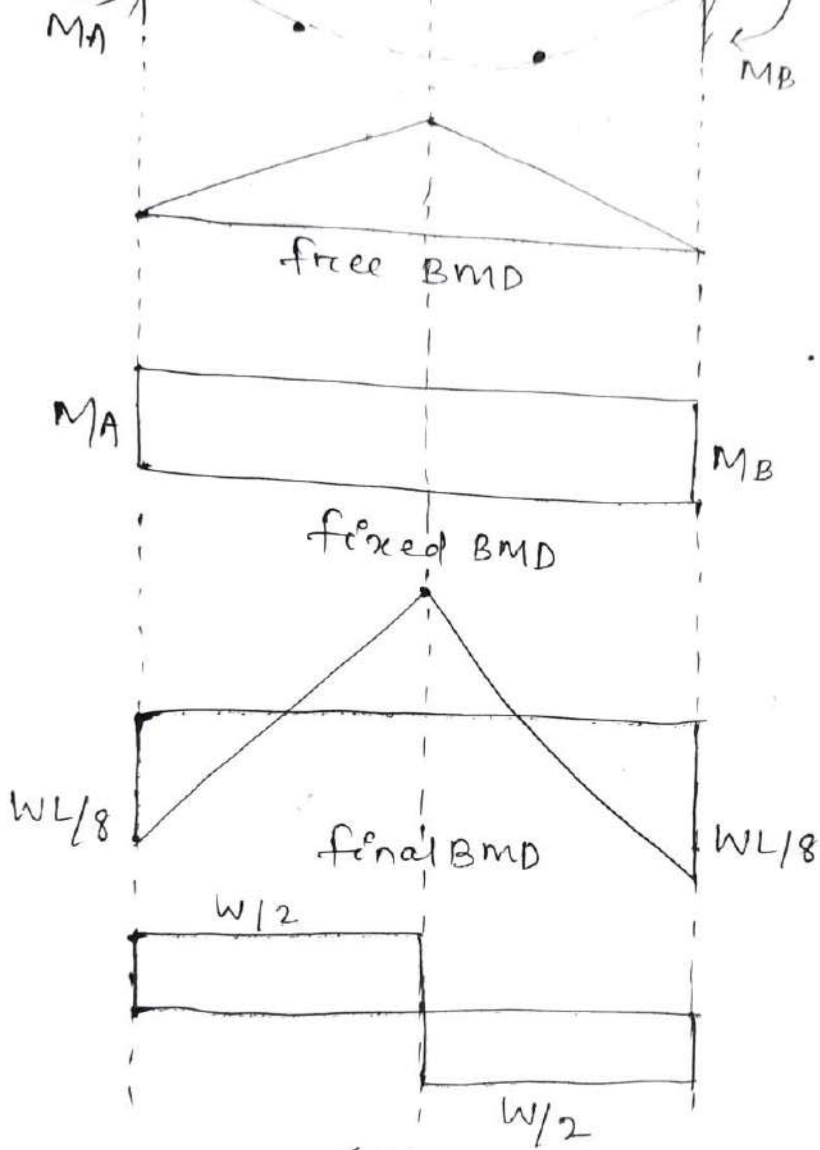
$$\boxed{\frac{A}{EI} = \frac{A_s}{EI} - \frac{A_f}{EI} = 0}$$

where, A_s = Area of free BMD.
A_f = Area of fixed BMD.

(ii) moment of area of M/EI diagram abo
support is zero.

$$\boxed{\frac{A_s}{EI} \times X_s + \frac{A_f}{EI} \times X_F = 0}$$

(iii) for beam of constant 'EI', the C.G.
free B.M.D and C.G of fixed BMD wil
equidistant from the same support.



SFD

Slope at fixed ends A and B

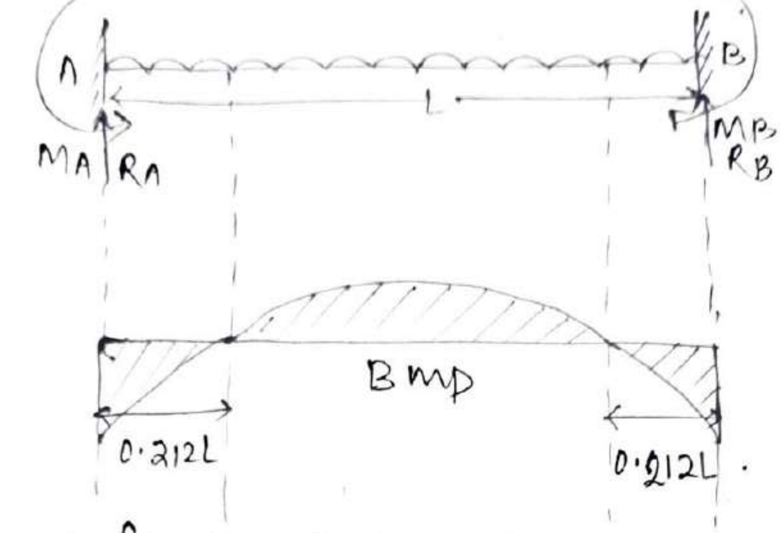
$$\theta_A = \theta_B = 0$$

Maximum central deflection at C

$$y_{max} = y_c = \frac{1}{4}\left(\frac{WL^3}{48EI}\right)$$

Maximum +ve BM = $\frac{WL}{8}$ (sagging)

Maximum -ve BM = $WL/8$ (hogging)



Slopes at fixed end A and B.

$$\theta_A = \theta_B = 0$$

Maximum central deflection

$$Y_{max} = y_c = \frac{1}{5}\left(\frac{wl^4}{384EI}\right) = \frac{wl^4}{384E}$$

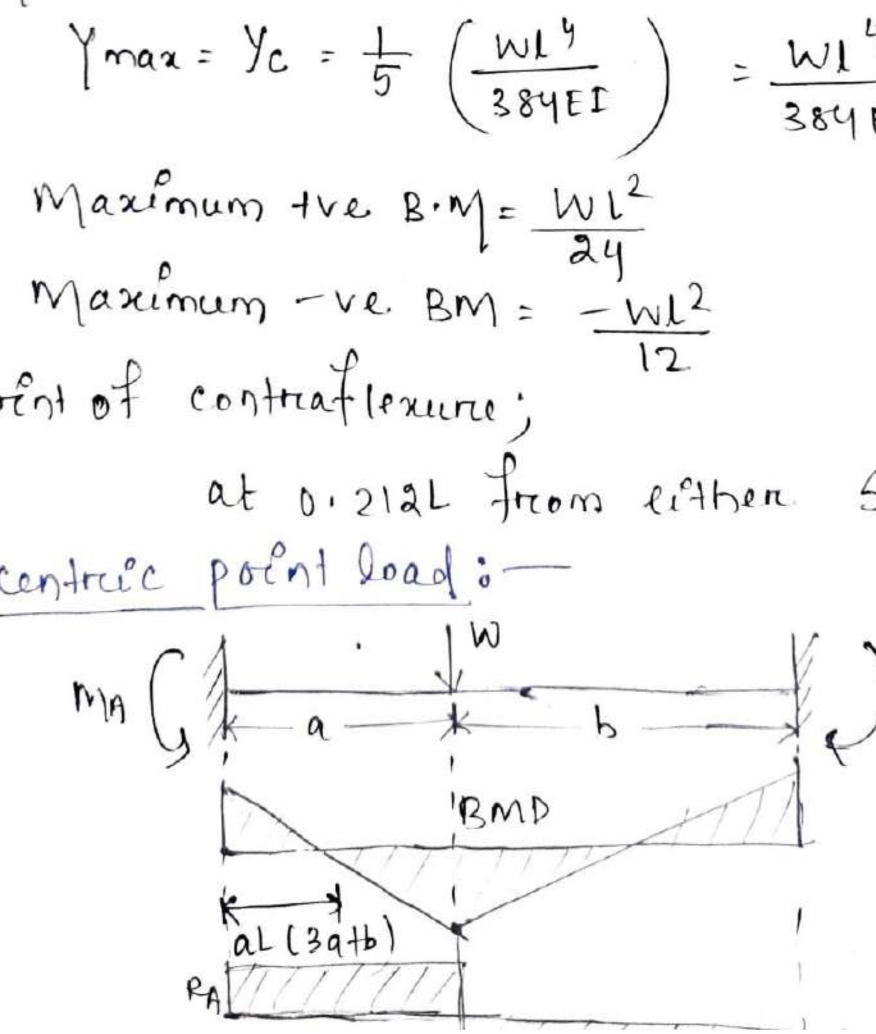
$$\text{Maximum +ve B.M} = \frac{wl^2}{24}$$

$$\text{Maximum -ve BM} = \frac{-wl^2}{12}$$

Point of contraflexure;

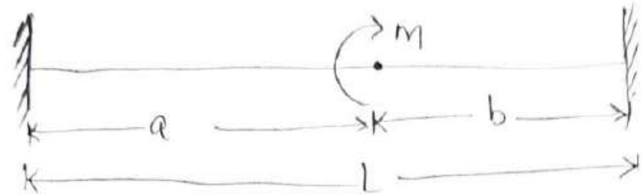
at 0.212L from either S

Eccentric point load:—



$M_B = \frac{Wa^2 b}{L^2}$ (hogging)

Couple :



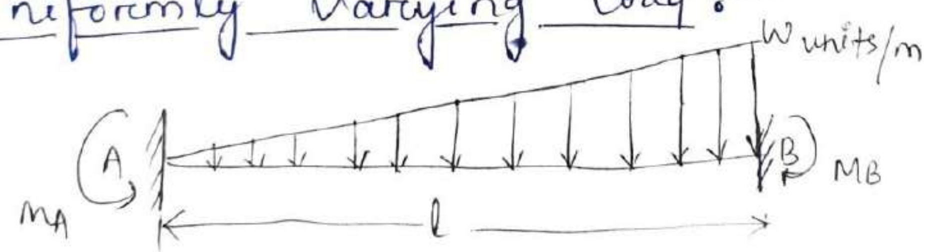
Moment at fixed end B

$$M_B = \left(\frac{Ma}{L^2}\right)(2b-a) \text{ (hogging)}$$

Moment at fixed end A

$$M_A = \left(\frac{Mb}{L^2}\right)(2a-b) \text{ (sagging)}$$

Uniformly varying Load :-



Moment at fixed end A.

$$M_A = \frac{WL^2}{30} \text{ (hogging)}$$

Moment at fixed end B.

$$M_B = \frac{WL^2}{20} \text{ (hogging)}$$

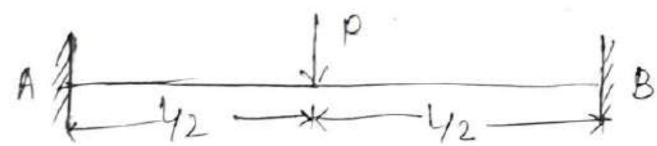
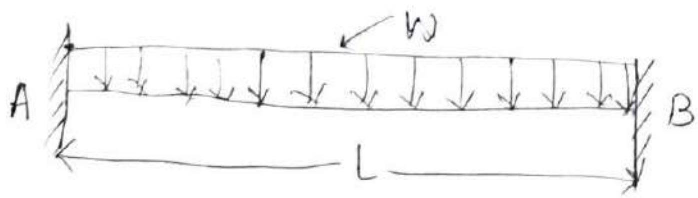
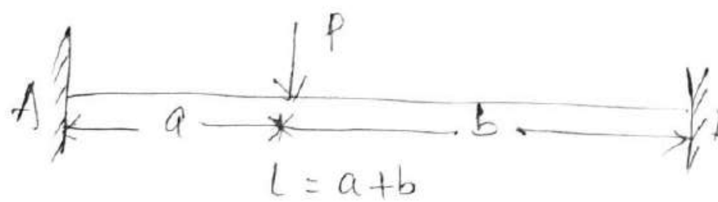
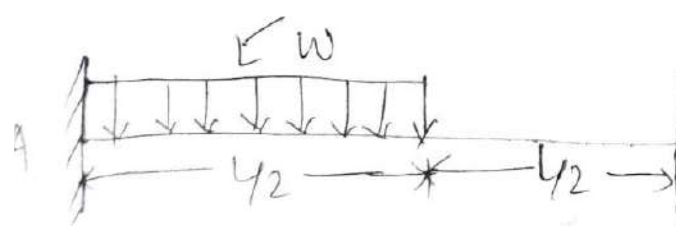
Sinking of Support :-

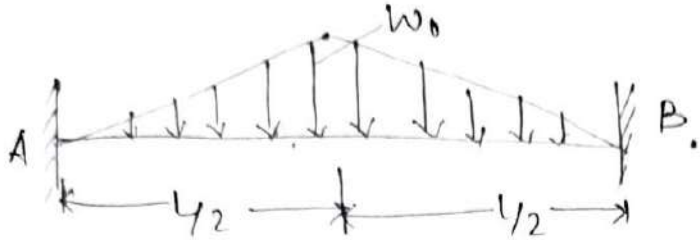
Bending moment at non-rigid end B

$$M_B = \frac{6EI\delta}{L^2} \text{ (sagging)}$$

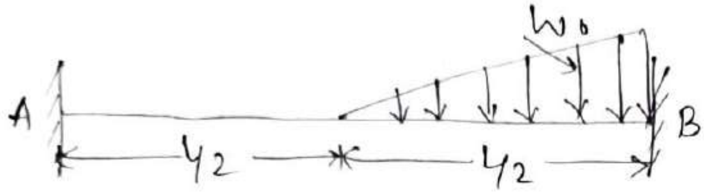
Rotation of Supports:-

At support 'B' anticlockwise rotation 'θ' r
is applied.

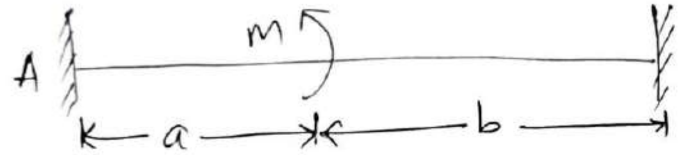
Type of loading	fixed end mo	
	FEM_{AB}	
	$+\frac{PL}{8}$	$-$
	$+\frac{WL^2}{12}$	$-$
	$-\frac{Pab^2}{L^2}$	$-\frac{P\ldots}{L\ldots}$
	$\frac{11}{192}WL^2$	$-\frac{5\ldots}{19\ldots}$



$+\frac{5}{96} w_0 l^2$



$+\frac{7}{960} w_0 L^2$



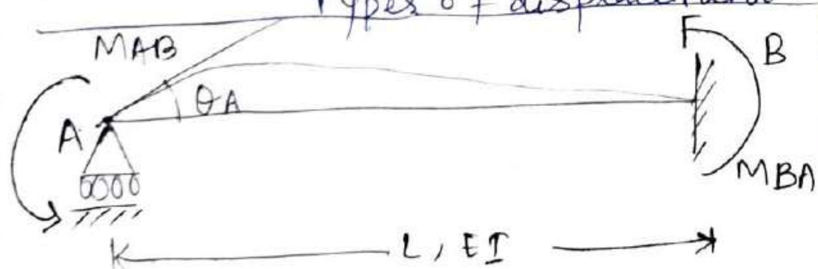
$+\frac{M}{l^2} b(b-2a$

$L = a+b$

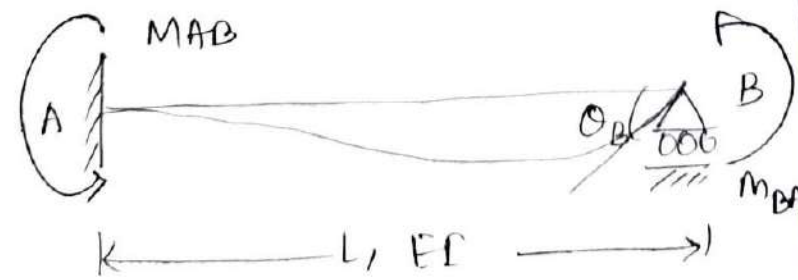
Types of displacement

END M

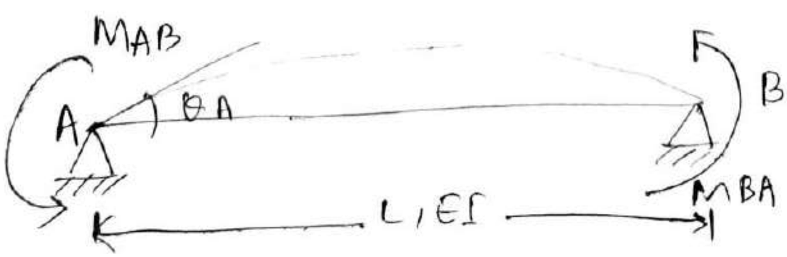
+FEMA



$+\frac{4EI\theta_A}{L}$

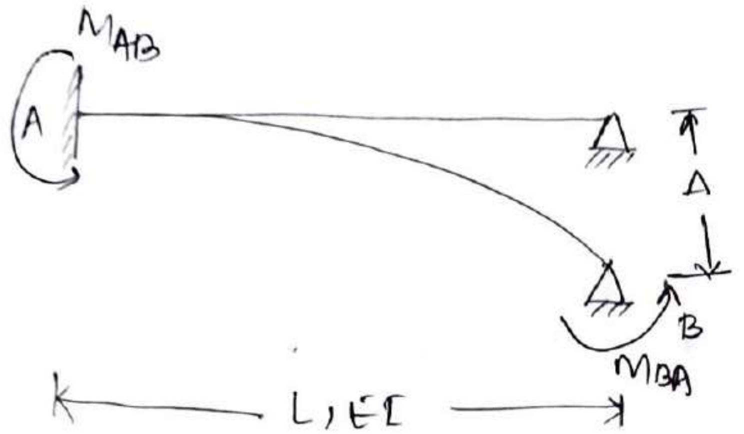


$+\frac{2EI\theta_B}{L}$



$+\frac{3EI\theta_A}{L}$

$$+\frac{6EI\Delta}{L^2}$$



$$+\frac{3EI\Delta}{L^2}$$

- Most Structures are made of several tr
together to form a space framework. Each
those loads which act in its plane and m
as a two-dimensional structure.

- Bolted or welded connections are ass
pinned together. forces acting at the m
reduce to a single force and no couple
force members are considered.

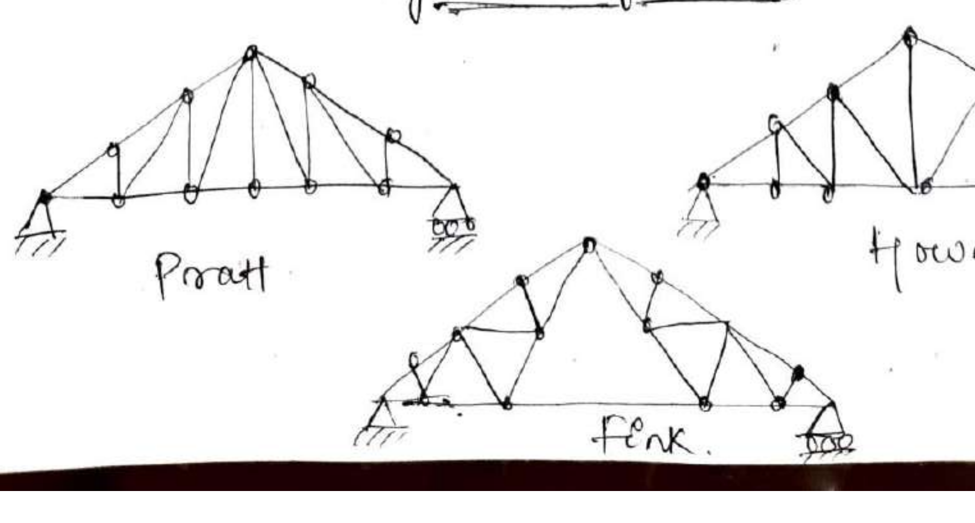
- when forces tend to pull the member a
tension. when the forces tend to compre
it is in compression.

Truss:-

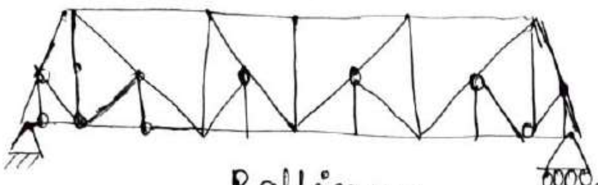
Members of a truss are slender a
of supporting large lateral loads. Loads
applied at the joints.

→ weights are assumed to be distribute
→ External distributed loads transferre
via stringers and floor beam.

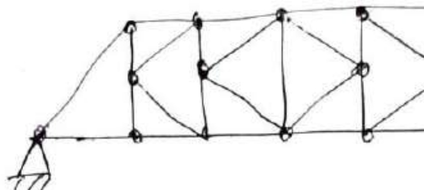
Typical roof trusses.



Pratt Howe War

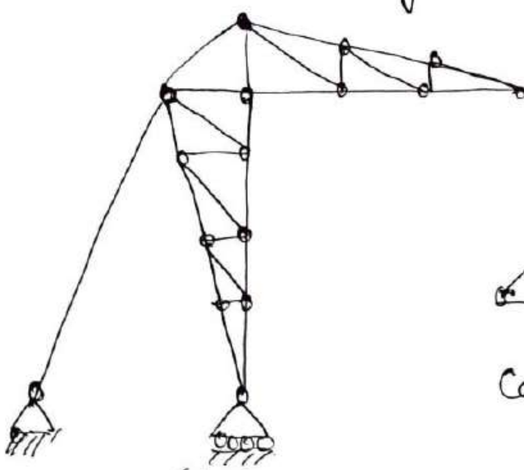


Baltimore

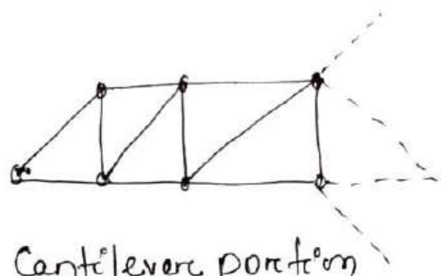


K truss

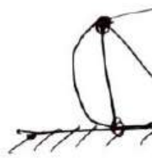
Other types of trusses :-



Stadium.



Cantilever portion of a truss.

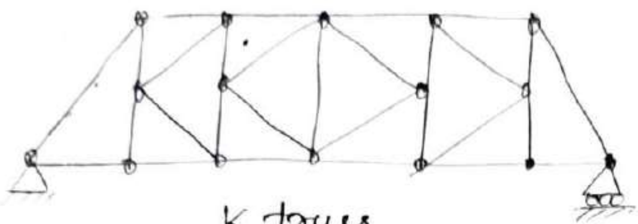


Simple trusses :-

- A rigid truss will not collapse under the
  of a load.
- A simple truss is constructed by succe
  two members and one connection to the b
  triangular truss.
- In a simple truss, $\boxed{m = 2j - 3}$ where m
  total number of members and j is the n
  joints.

Baltimore

M = 45 , j = 24



← Simple tr

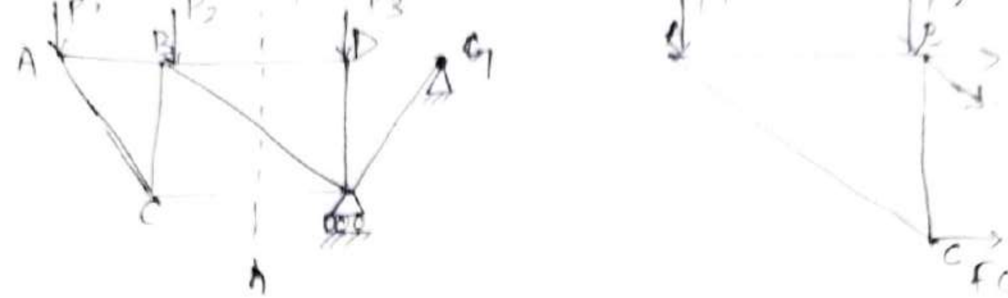
K truss

m = 29 , j = 16 .

- A simple truss is constructed by success two members and one connection to th triangular truss.
- In a simple truss, $m = 2j - 3$ where m is t number of members and j is the number

<u>Analysis of trusses by the method of joi</u>

- Dismember the truss and create a free for each member and pin.
- The two forces exerted on each member are have the same line of action, and opp
- Forces exerted by a member on the pins at its ends are directed along the member and opposite.
- Conditions of equilibrium on the pins equations for 2j unknowns. For a sim $2j = m + 3$. May solve for m member for 3 reaction forces at the supports.



- when the force is only one member or in a very few members are desired, the of sections works well.
- To determine the force in member BD, section through the truss as shown and free body diagram for the left side (or
- with only three members cut by the se equations for static equilibrium may to determine the unknown member for including F_{BD}.

Important Notes

- for a truss to be properly constrained:
- It should be able to stay in equilibrium combination of loading.
- Equilibrium implies both global equili internal equations equilibrium.

Note that if $2j > m + r$, the truss is mos partially constrained (and is unstable to cer

But $2j \geq m + r$, is no guarantee that truss

If $2j < m + r$, the truss can never be stati determinate.

structure effectively manages both com
tension, by spreading out the load from
way throughout its intricate structure.
that no one part of the structure is carr
disproportionate amount of weight.

- Uses materials effectively :-

While the truss bridge has many parts,
parts to make up its structure - its use
is extremely effective. Materials such as
and steel are all utilised to their highest
and every piece plays a role. the buil
large truss bridge can be a very econom
when compared to other bridge designs

- With stands extreme conditions :-

Where other bridges such as beam and
may not be a ~~viable~~ viable option, truss
into their own. they are able to span
and often used in precarious locations
ravines between mountain tops. you'll
see truss bridges in use throughout mo
areas to carry railways.

- Roadways built on to the structure :-

Unlike other bridge designs, the truss b
to carry its roadway on its structure
can be carried above (deck truss), al
(through truss) or on a bottom truss, w
below the major truss structure.

400 metres.

Dis advantages :-

- Requires a lot of space :-

the structure of a truss bridge is large by d[illegible] the inter connecting triangular components ne[illegible] be large in order to bear and distribute h[illegible] loads. this means that in certain restricted s[illegible] the truss bridge may not be the best option

- High maintenance costs :-

the truss bridge uses a lot of parts. Each of th[illegible] are relatively light, and used effectively with[illegible] the design, which means that if you are buil[illegible] a huge truss bridge it is economically sen[illegible] However the maintainance costs of so many p[illegible] can be expensive. A truss bridge, like an[illegible] load-bearing structure, will require regula[illegible] detailed maintenance. ~~so many~~

- Skilled labour is required :-

Truss bridges are intricate, complex. Each and every piece needs to fit perfectl[illegible] in order to perform its function, and anythi[illegible] less will mean that the bridge simply does[illegible] hold a load. A truss bridge requires d[illegible] engineering and specialist construction, th[illegible] not come cheap..

maybe other bridge options

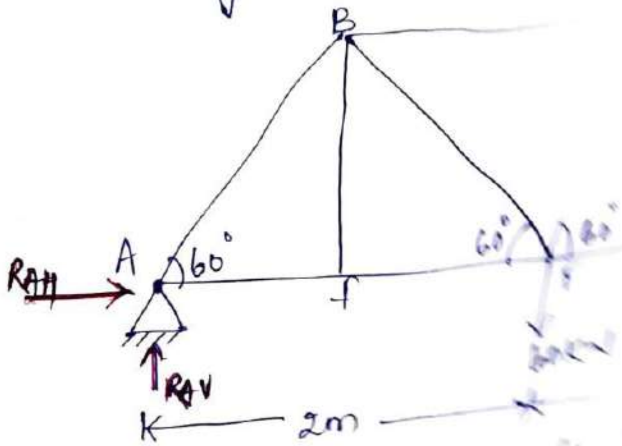
beam bridges, which might

If your landscape cannot

Problems on truss by joint

Q find the force in all the members

shown by the method of joint



Before that, you

- How to draw
- All the

  ΣF_y

- All the

. 1st draw

members

. We use

and

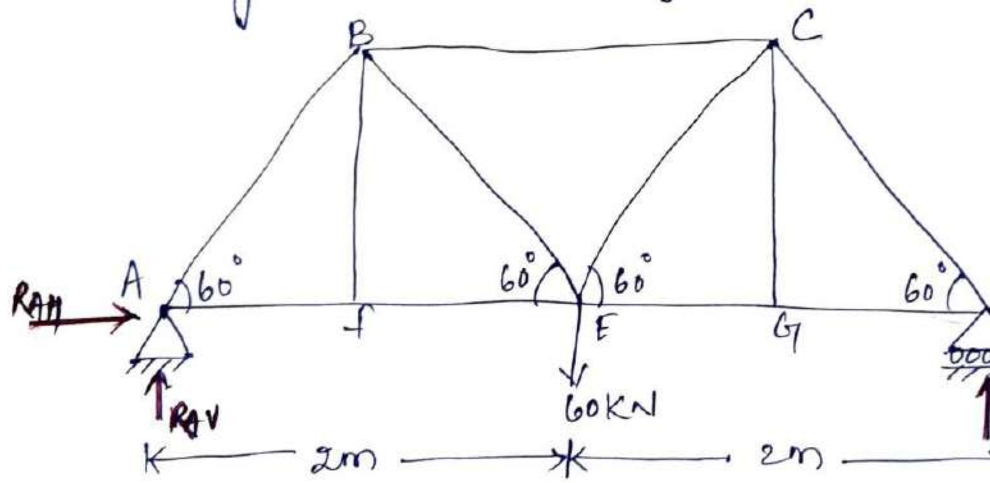
hinge

F

may be necessary to cope with the wei
maybe other bridge options such as
beam bridges, which might be more
If your landscape cannot support a

Problems on truss by joint method:-

Q find the force in all the members of
shown by the method of joint.



Before that, you need to know.

- How to draw free body diagram (F
- All the horizontal & vertical for
  $\Sigma fy = 0$, $\Sigma fx = 0$
- All the momement, $\Sigma M = 0$

- 1st draw FBD of the joint where t
  members are connected. i.e. point
- we need to remove supports (hinge
  and show the reactions (unknown)
  hinged → 2 unknown support.
  Roller → 1 unknown support.

$\xrightarrow{+} \Sigma F_x = 0$,

$$\boxed{R_{AH} = 0}$$

$\curvearrowright + \Sigma M_A = 0$, (All the passing force at point $\underline{A} = 0$)

$$60 \times 2 - R_D \times 4 = 0$$

$$-R_D \times 4 = -120$$

$$\Rightarrow \boxed{R_D = 120/4 = 30KN}$$

$+\uparrow \Sigma F_y = 0$,

$$R_{AV} + 30 - 60 = 0$$

$$\boxed{R_{AV} = 30KN}$$